# Protecting Quantum Criticality — when is the system conducting/insulating?

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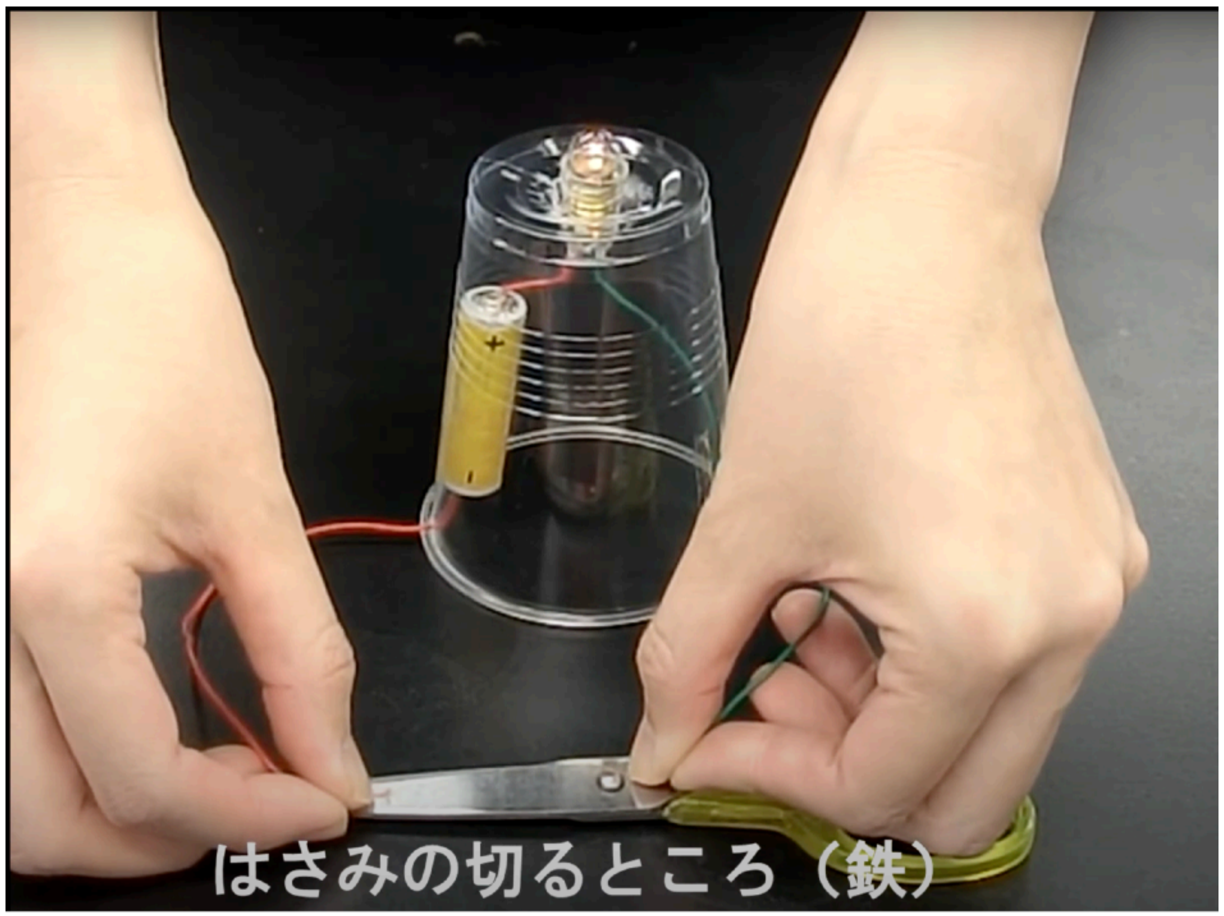
# Fundamental Classification

[Sendai Science Museum Video for Grade 3]

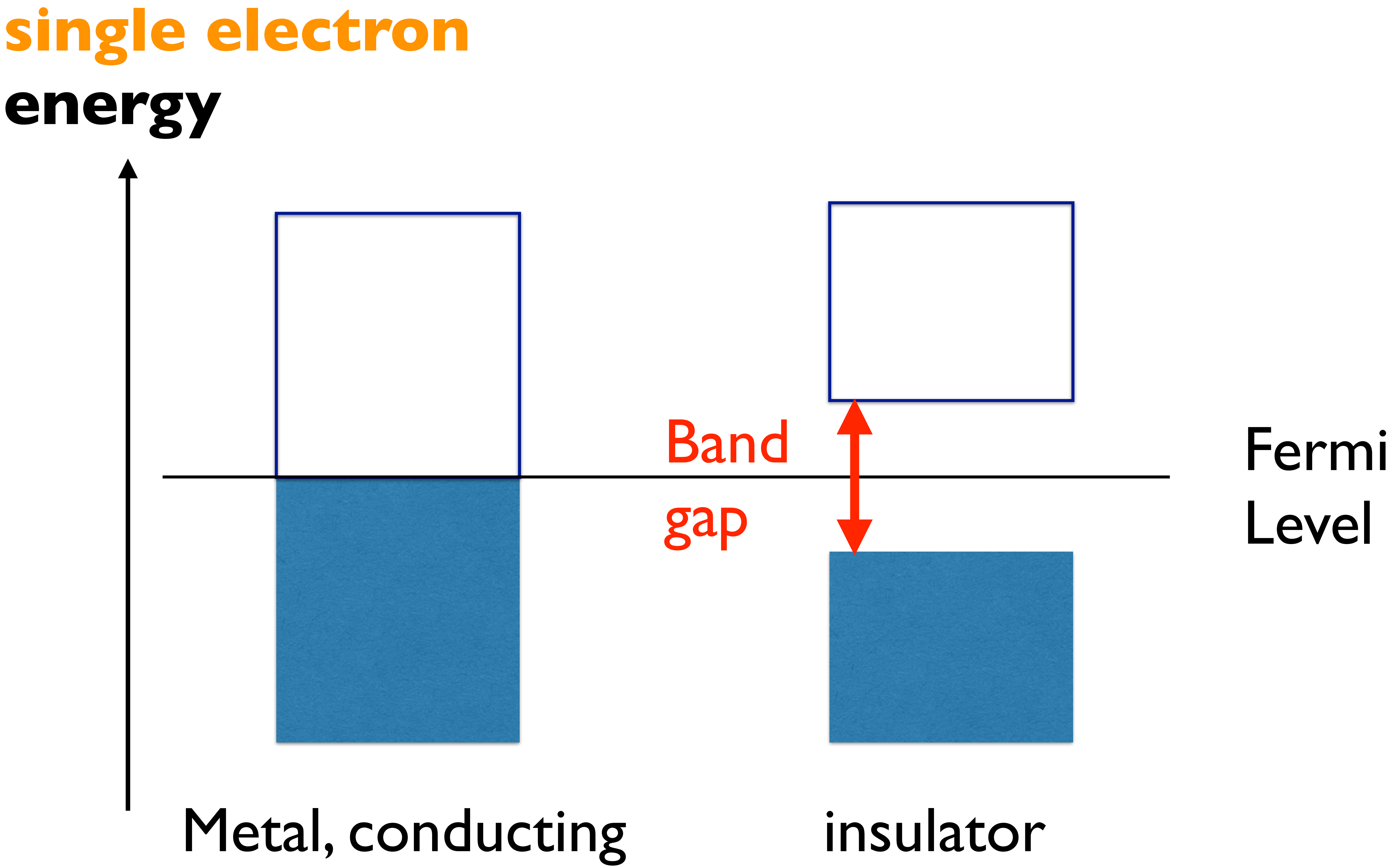




Conductor

Insulator

# Free Electron Systems

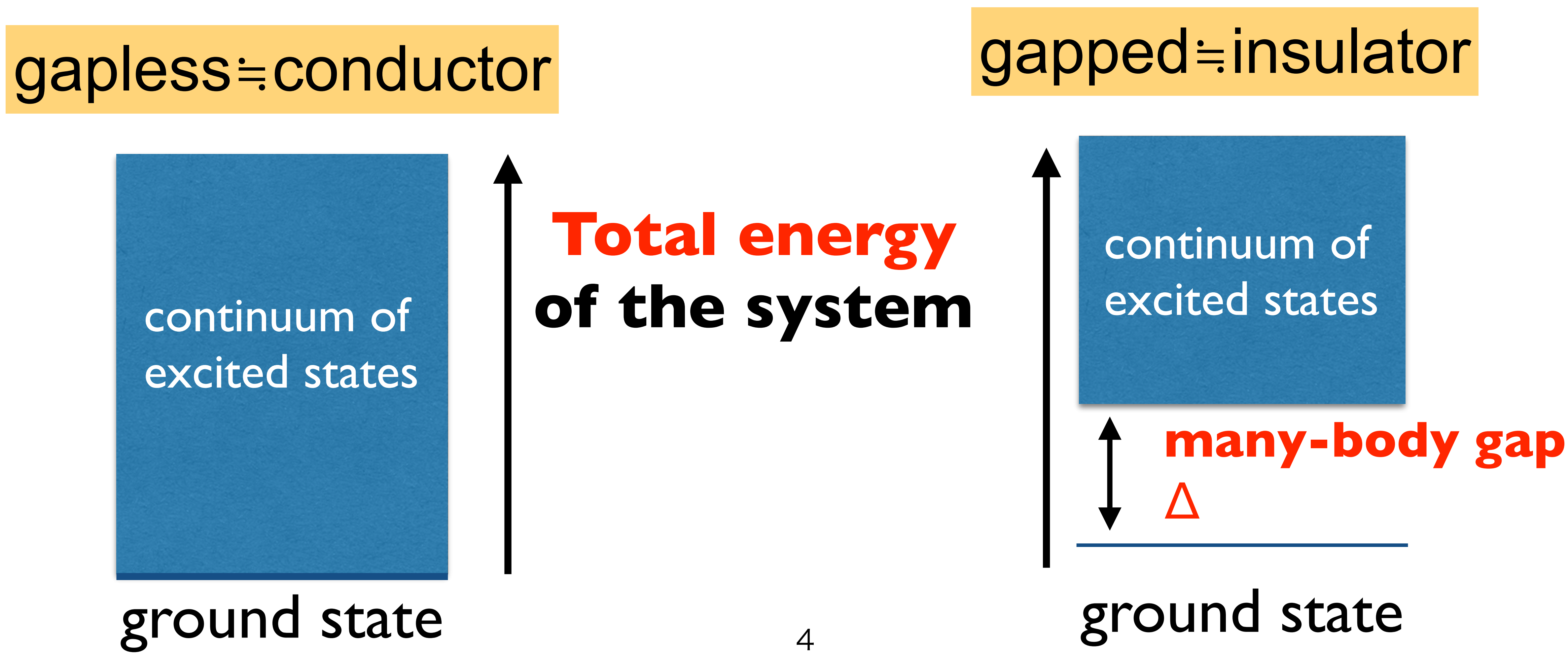


# Interacting Many-Electrons

We can no longer describe physics in terms of single electron

We cannot apply the "band picture" in general

We can still characterize the system in terms of many-body excitation gap!



# Different Cultures…



not just different languages, but
different backgrounds, different points of view

# Solid State Physics

Electrons form conducting metal by default
(e.g. in free space)

Some mechanism is needed for electrons to
form an insulator by opening a gap
(CDW instability, etc.)

# Statistical Mechanics

Many-body gap Δ = inverse "correlation time"
(in imaginary time formalism)

$$\langle \mathcal{O}(\tau)\mathcal{O}(0)\rangle \propto e^{-\Delta\tau}$$

Correlation length is often proportional to 1/Δ

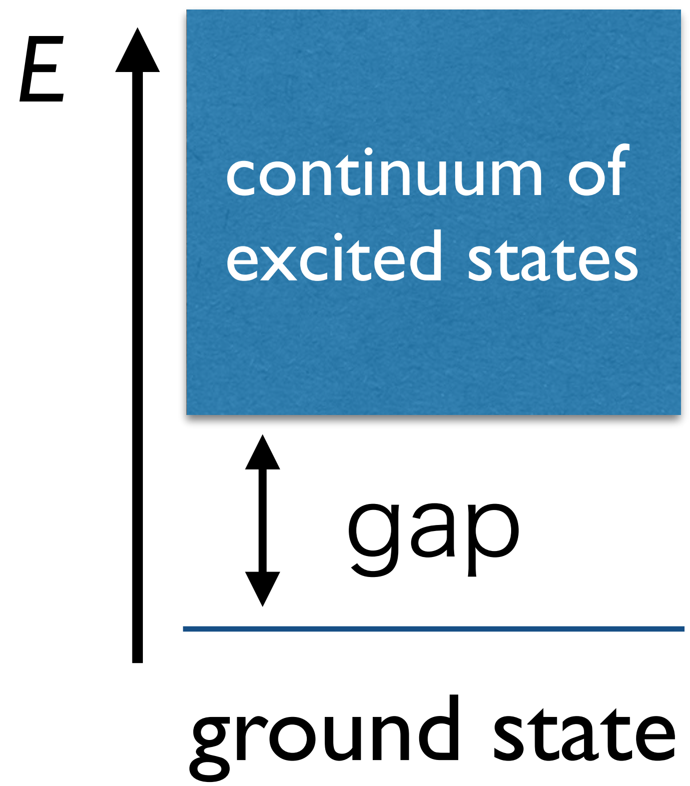
gapped (≒ insulator) ⇔ finite correlation length
off-critical

gapless (≒ conductor) ⇔ ∞ correlation length
critical

# Transverse-Field Ising Model

$$\mathcal{H} = -\sum_{\langle j,k \rangle} \sigma_j^z \sigma_k^z - g \sum_j \sigma_k^x$$

gapped (off-critical)




TFIM in $d$ dimensions
↕
Classical Ising Model in $d$+1 dimensions

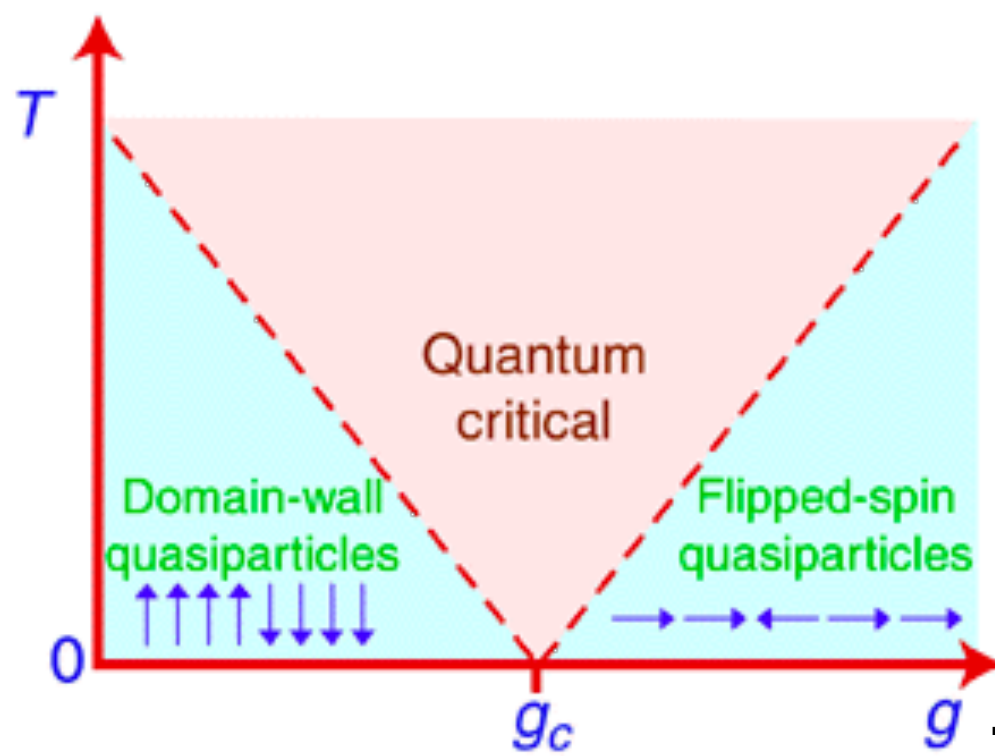
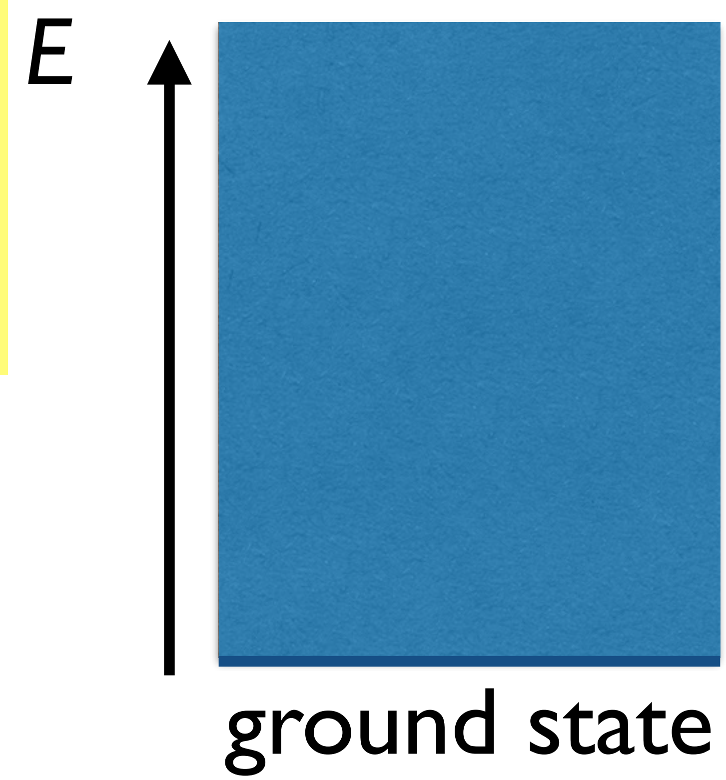



fig. by Subir Sachdev

gapless (critical)



# Stat Mech Point of View

Gapless, critical systems (≒ conductor)
are special, usually requires fine-tuning of parameter(s)

However, there are many gapless, critical systems
in condensed matter physics which apparently
do NOT require any fine-tuning
(Metals, phonons, …..)

Why?
There must be some **mechanism to protect quantum criticality** (gaplessness)….

# General Principles?

Symmetries of the model

$$\mathcal{H} = \sum_{\langle j,k \rangle} \left[ -t \left( c_j^\dagger c_k + c_k^\dagger c_j \right) + V n_j n_k \right]$$

U(1) symmetry

$$c_j \rightarrow e^{i\theta} c_j$$

$$c_j^\dagger \rightarrow e^{-i\theta} c_j^\dagger$$

$$n_j \equiv c_j^\dagger c_j \rightarrow n_j$$

particle number

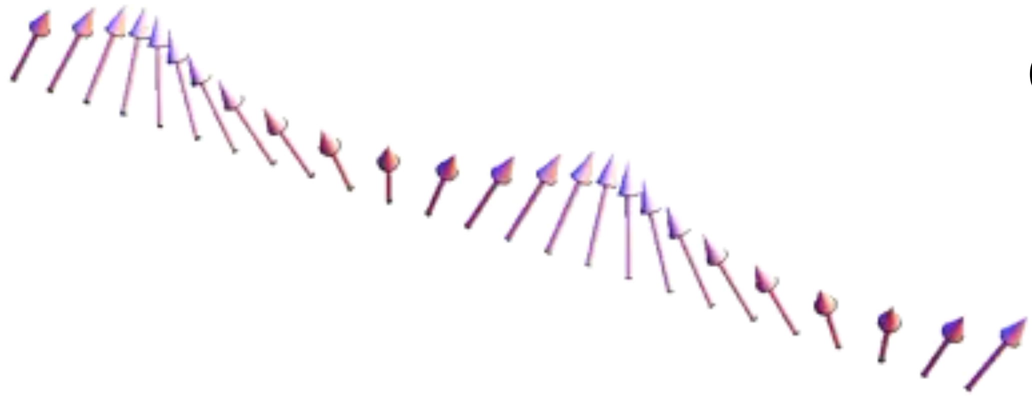
$$N \equiv \sum_j n_j$$

conserved

Noether's theorem

Can we say something about the energy spectrum?

# Nambu-Goldstone Theorem



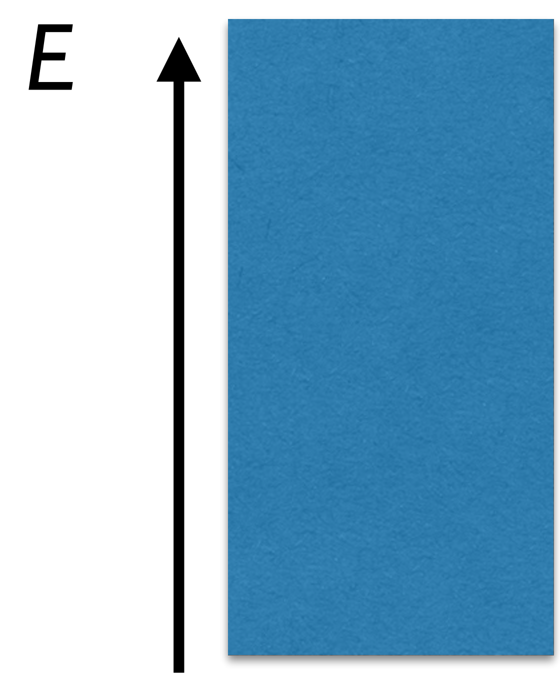
e.g. spin waves

Spontaneous breaking of a continuous symmetry (e.g. U(1))

"slow twist"

Gapless excitations

gapless (critical)

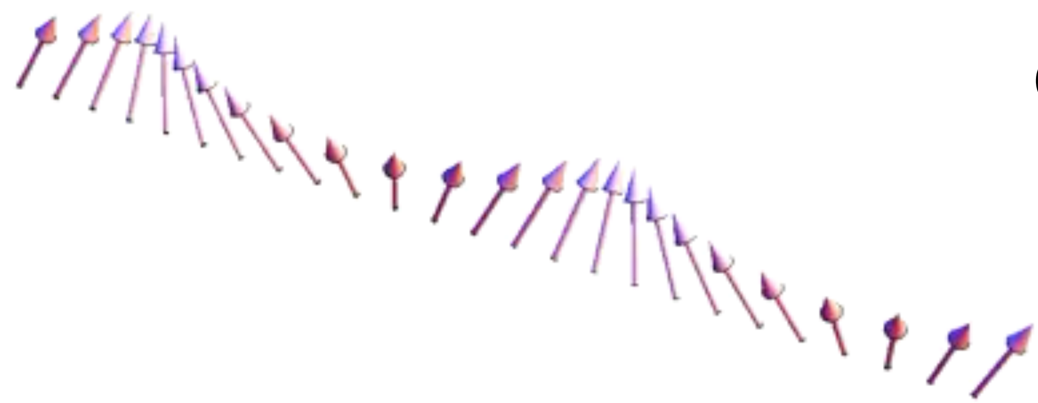



There are many gapless systems without a SSB (metals, etc.), however.

Any other mechanism for gaplessness?

**yes, if there is also a lattice translation invariance**

# Nambu-Goldstone Theorem



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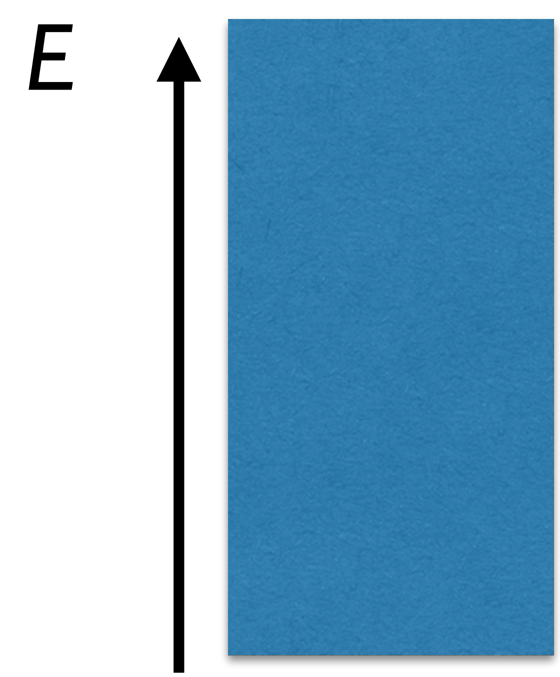


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# Lieb-Schultz-Mattis Theorem in 1D

Lieb-Schultz-Mattis 1961, M.O.-Yamanaka-Affleck 1997,…

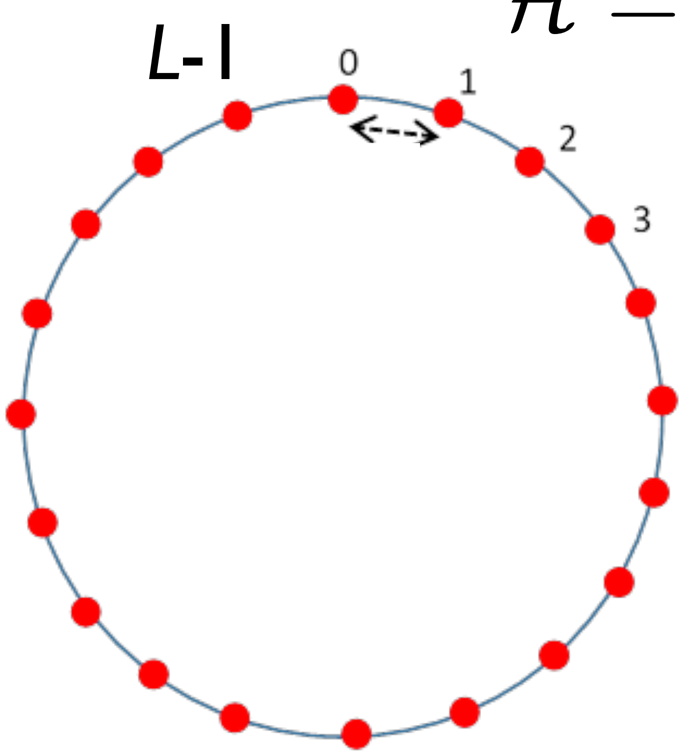
Number of particles: conserved ← U(1) symmetry

Lattice translation symmetry +

spatial inversion or time reversal symmetry

e.g. 1D spinless Hubbard model with periodic b.c. $c_L \equiv c_0$

$$\mathcal{H} = -t \sum_{j=0}^{L-1} \left( c_{j+1}^\dagger c_j + c_j^\dagger c_{j+1} \right) + V \sum_{j=0}^{L-1} n_j n_{j+1}$$




Lattice translation $\mathcal{T}$ $\quad \mathcal{T} c_j \mathcal{T}^{-1} = c_{j+1}$

Translation inv. $\quad [\mathcal{T}, \mathcal{H}] = 0$

# LSM Variational Argument

Ground state $\mathcal{H}|\Psi_0\rangle = E_0|\Psi_0\rangle$

(very complicated — we don't need to know it exactly
its EXISTENCE is enough!)

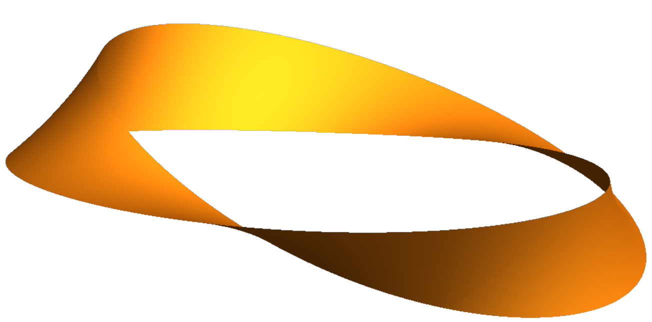
$e^{i\theta N} = e^{i\theta \sum_j n_j}$ global U(1) transformation $c_j \to e^{i\theta} c_j$

"Slow twist" (NOT symmetry) $\mathcal{U} \equiv \exp\left(\sum_j \frac{2\pi i j}{L} n_j\right)$

$$\mathcal{U}^\dagger c_j \mathcal{U} = \exp\left(\frac{2\pi i j}{L}\right) c_j$$

consistent with PBC $c_L \equiv c_0$

$$\mathcal{U}^\dagger c_0 \mathcal{U} = \exp\left(\frac{2\pi i 0}{L}\right) c_0 = c_0$$



$$\mathcal{U}^\dagger c_L \mathcal{U} = \exp\left(\frac{2\pi i L}{L}\right) c_L = c_L$$

# LSM Variational Argument

$$\mathcal{U}^\dagger \mathcal{H} \mathcal{U} = -t \sum_{j=0}^{L-1} \left( e^{-2\pi i/L} c_{j+1}^\dagger c_j + e^{2\pi i/L} c_j^\dagger c_{j+1} \right) + V \sum_{j=0}^{L-1} n_j n_{j+1}$$

$$\mathcal{H} = -t \sum_{j=0}^{L-1} \left( c_{j+1}^\dagger c_j + c_j^\dagger c_{j+1} \right) + V \sum_{j=0}^{L-1} n_j n_{j+1}$$

$$\mathcal{U}^\dagger \mathcal{H} \mathcal{U} - \mathcal{H} = t \frac{2\pi i}{L} \sum_j \boxed{\left( c_{j+1}^\dagger c_j - c_j^\dagger c_{j+1} \right)} \rightarrow \text{expectation value vanishes}$$

$$+ \boxed{t \left( \frac{2\pi}{L} \right)^2 \sum_j \left( c_{j+1}^\dagger c_j + c_j^\dagger c_{j+1} \right)} + O(\frac{1}{L^2})$$

$$\langle \Psi_0 | \left( \mathcal{U}^\dagger \mathcal{H} \mathcal{U} - \mathcal{H} \right) | \Psi_0 \rangle = O(\frac{1}{L})$$

$\mathcal{U}|\Psi_0\rangle$ is a low-energy state

# Does it mean anything?

$\mathcal{U}|\Psi_0\rangle$ could be (almost) identical to $|\Psi_0\rangle$

Are they different?

"filling factor"
(particle # / site)

$$\mathcal{T}|\Psi_0\rangle = e^{iP_0}|\Psi_0\rangle$$

$$\mathcal{U}^\dagger \mathcal{T} \mathcal{U} = e^{2\pi i \sum_j n_j / L}\mathcal{T} = e^{2\pi \nu i}\mathcal{T}$$

$$\nu = \frac{\sum_j n_j}{L} = \frac{N}{L}$$

$$\mathcal{U} \equiv \exp\left(\sum_j \frac{2\pi i j}{L} n_j\right)$$

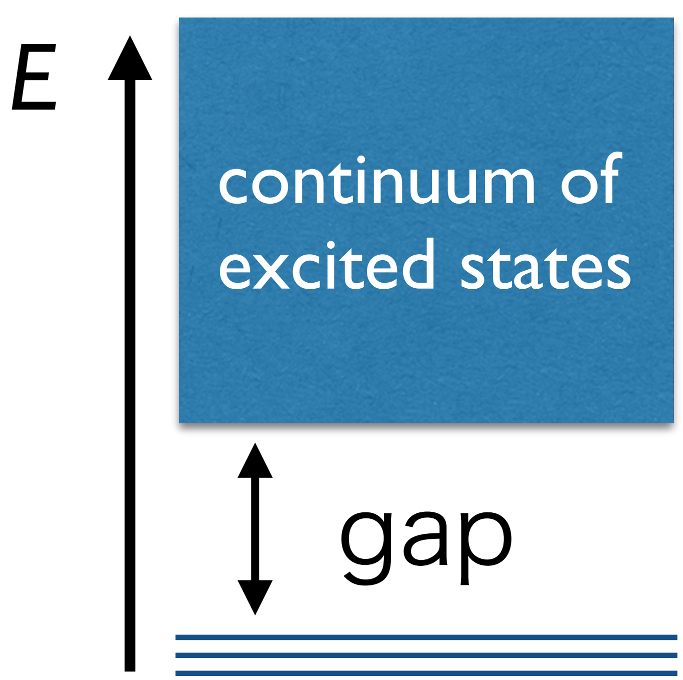
$\mathcal{U}|\Psi_0\rangle$ is a low-energy state different from $|\Psi_0\rangle$ if ν is NOT integer!

$$\mathcal{T}\left(\mathcal{U}|\Psi_0\rangle\right) = e^{iP_0 + 2\pi\nu i}\left(\mathcal{U}|\Psi_0\rangle\right)$$

# Consequences

If the filling factor $\nu = p/q$ is non-integer
($p$, $q$: coprimes)
there are $q$ independent low-energy states including the ground state

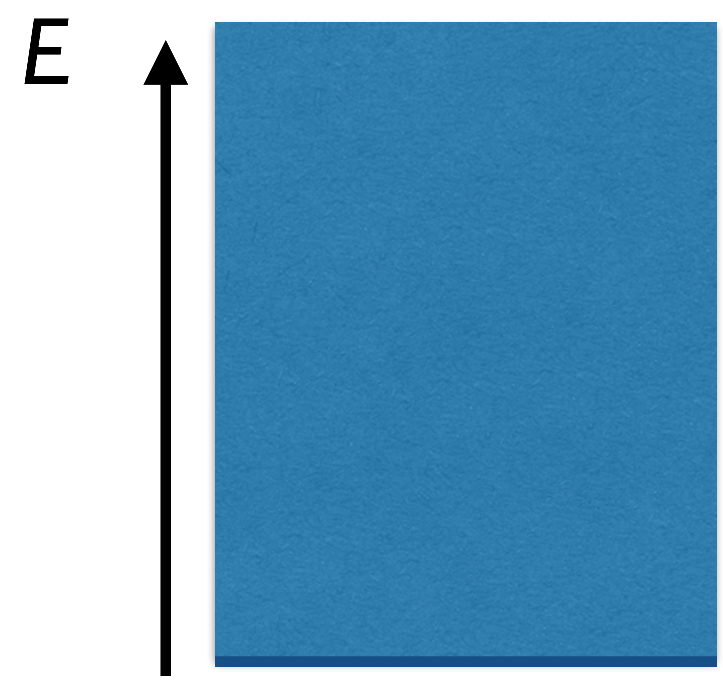
**gapped (off-critical)**




*q*-fold degenerate
ground state

OR

**gapless (critical)**




ground state

# Statement of LSM theorem

Quantum Many-Body System (in 1D) with

- global U(1) symmetry

AND

- lattice translation symmetry

WITH a fractional (non-integer) filling factor ν

⬇

- gapless excitations above the ground state

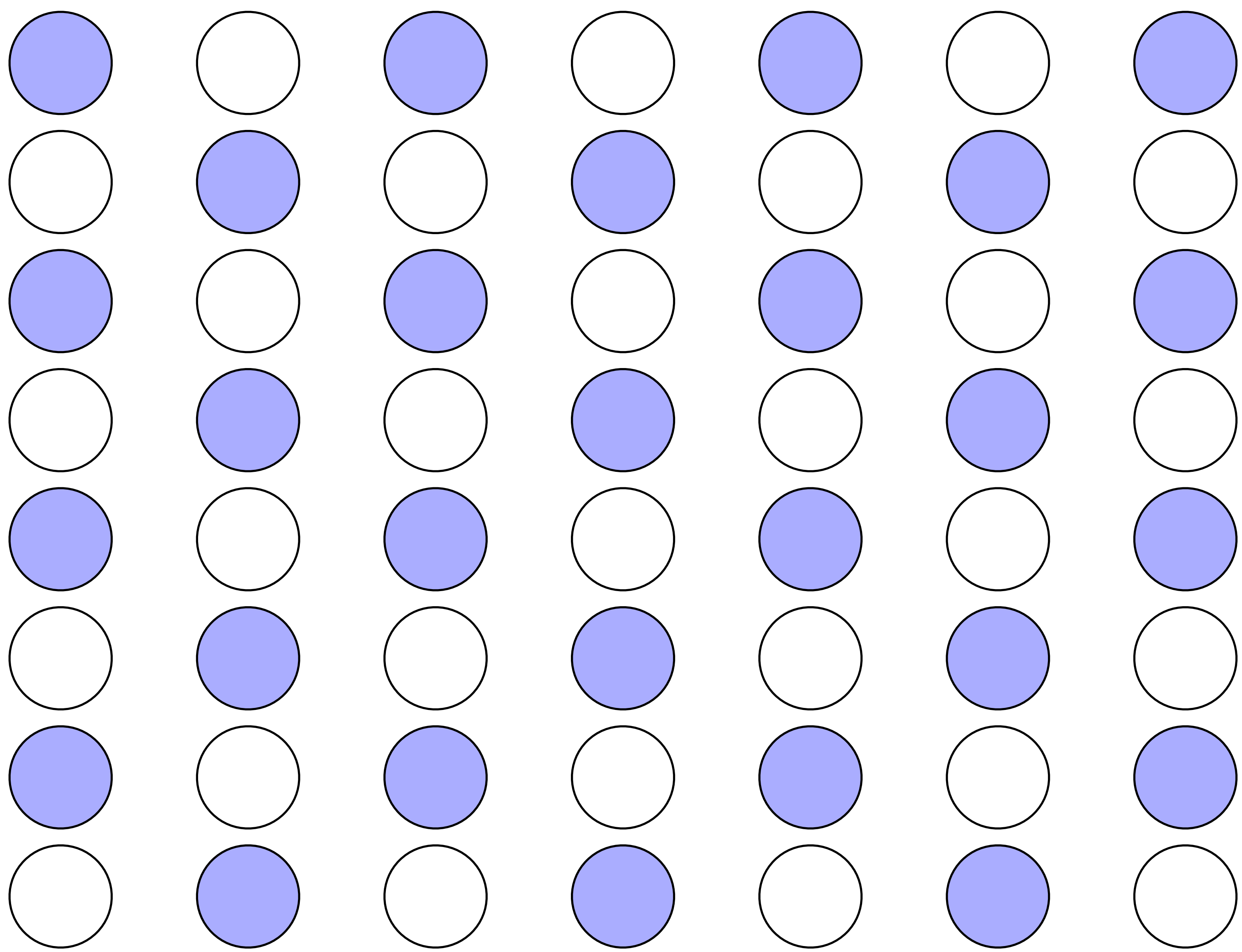
OR

- multiple, degenerate ground states below gap

~~- unique ground state below gap~~

~~“featureless (trivial) insulator”~~

**Intuitive picture for the LSM theorem: gapped phase needs the particles to be "locked", and the density of the particles must be commensurate with the lattice.**



1 particle/
unit cell
(= 2 sites)

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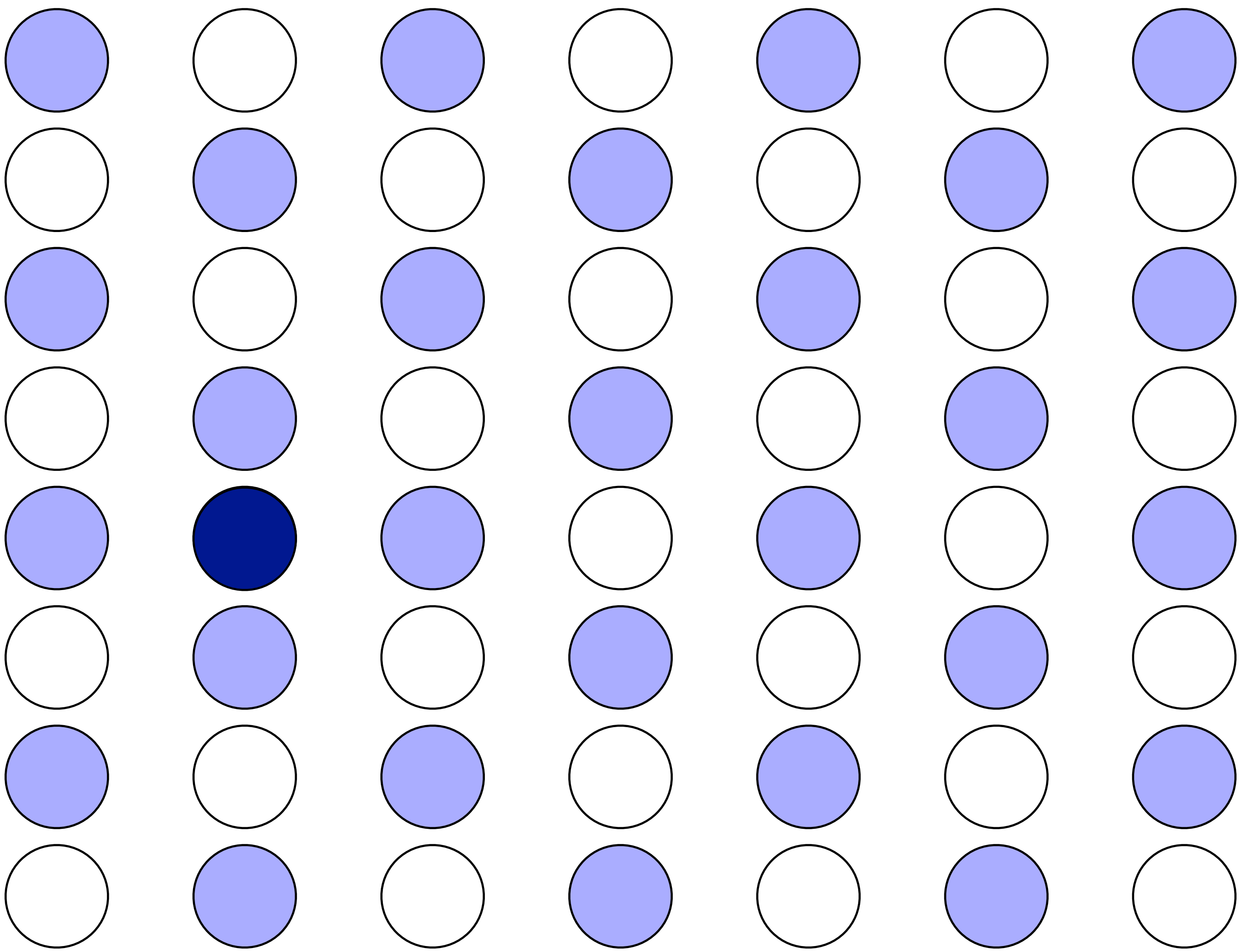
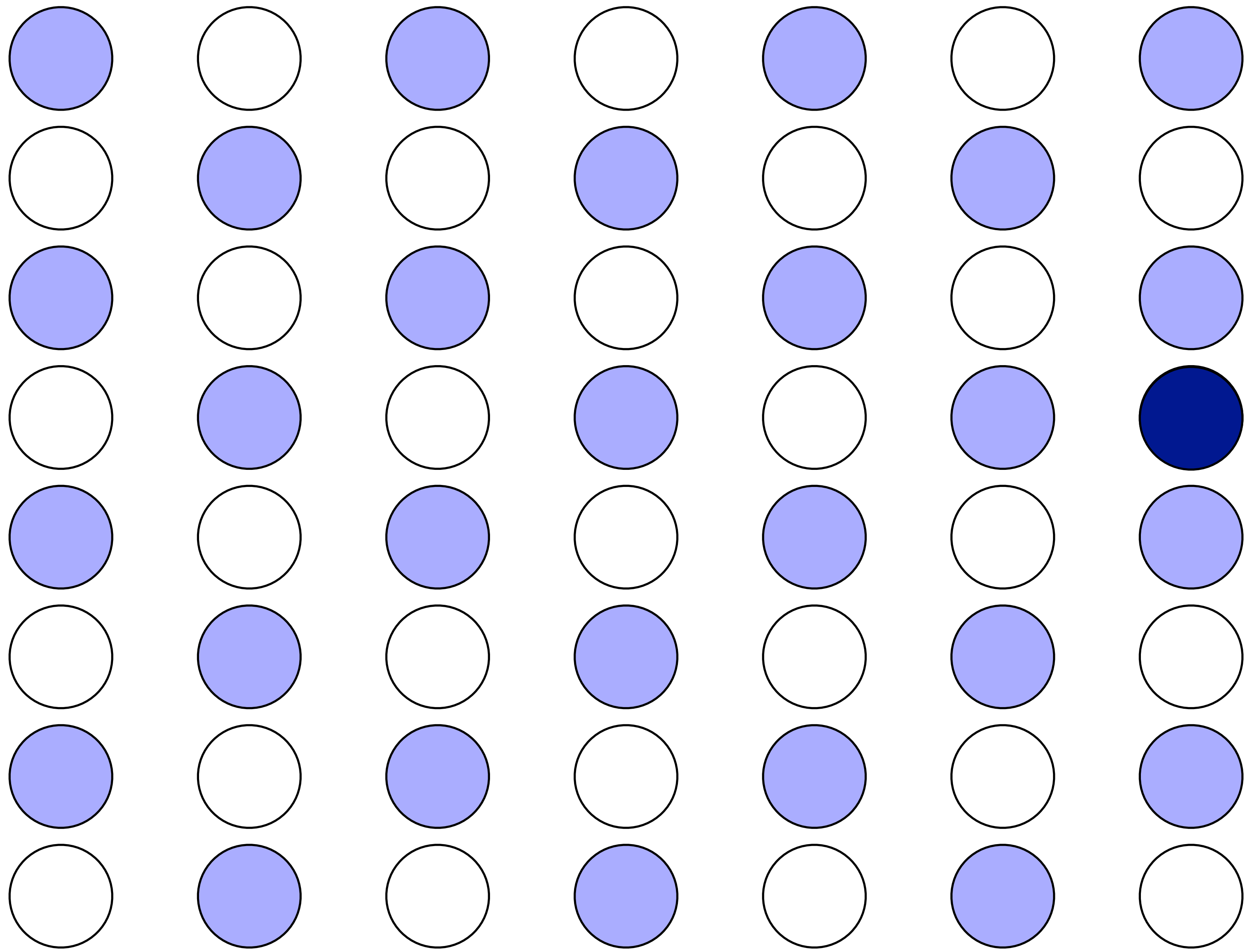


1 particle/ unit cell (= 2 sites)

add extra particles ("doping")

# Intuitive picture for the LSM theorem: gapped phase needs the particles to be "locked", and the density of the particles must be commensurate with the lattice.



1 particle/ unit cell (= 2 sites)

add extra particles ("doping")

mobile carriers

# LSM for Spinful Electrons

Yamanaka-M.O.-Affleck 1997

Typical model: Hubbard model at half-filling

$$\mathcal{H} = -t \sum_{j,\sigma} c^{\dagger}_{j+1,\sigma} c_{j,\sigma} + U \sum_{j} n_{j,\uparrow} n_{j,\downarrow}$$

Total electron filling (# per site): $\nu=1$ (integer)

→ LSM cannot be applied??

Nevertheless, $\nu_{\uparrow}=\nu_{\downarrow}=1/2$ (non-integer)

→ ground-state degeneracy with gap

OR gapless

But we don't know the nature of gapless excitations

# Charge Gap and Spin Gap

"Spin-charge separation"

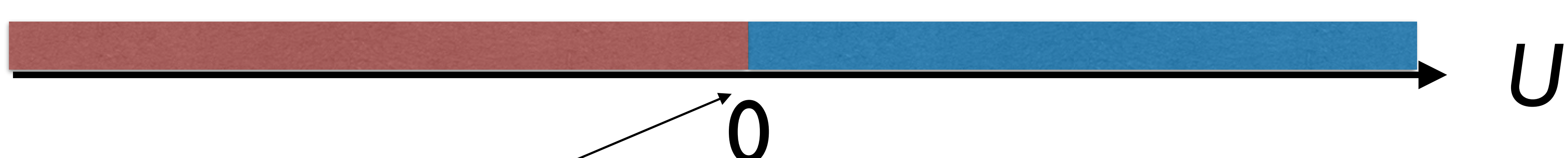
If the gapless excitations are genuinely spin excitations, the system is (electrically) insulator but spin conductor

If the gapless excitations are genuinely charge excitations, the system is (electrically) conductor but spin insulator

$$\mathcal{H} = -t \sum_{j,\sigma} c^\dagger_{j+1,\sigma} c_{j,\sigma} + U \sum_j n_{j,\uparrow} n_{j,\downarrow}$$ at half filling

Spin gap
gapless charge excitations
Luther-Emery Liquid

Charge gap
Mott insulator
gapless spin excitations




free electron metal

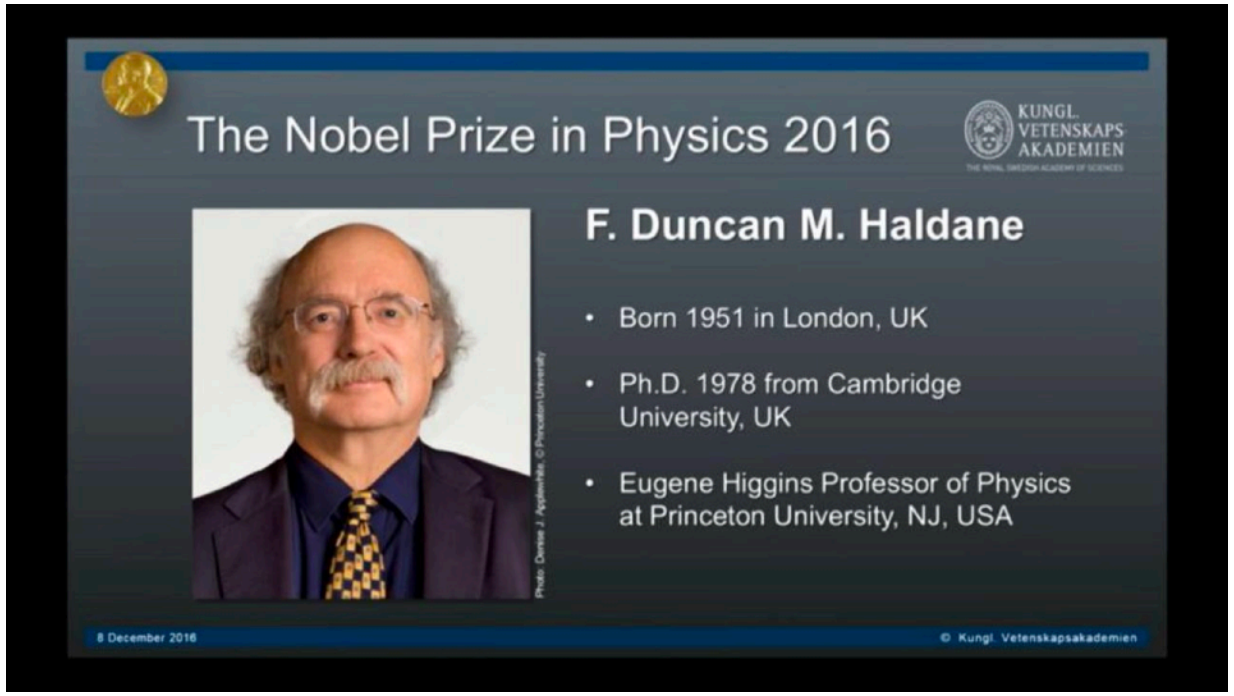
# Haldane "Conjecture" in 1981

$$\mathcal{H} = J \sum_j \vec{S}_j \cdot \vec{S}_{j+1}$$

S=1/2, 3/2, 5/2, .....

Gapless "Quantum Critical"

$$\langle \vec{S}_i \cdot \vec{S}_j \rangle \propto \left( \frac{1}{r} \right)^{\eta}$$




S=1, 2, 3.....

Non-vanishing excitation gap ("Haldane gap")

$$\langle \vec{S}_i \cdot \vec{S}_j \rangle \propto \exp\left( -\frac{r}{\xi} \right)$$

Against the "common sense" at the time ⇒ "conjecture"

# Spin System as Many Particles

Spin $S$: $S^z = -S, -S+1, \ldots, S-1, S$

M.O.-Yamanaka-Affleck 1997

Identify, say, $S^z = -S$ state as "vacuum"

increase $S^z$ by 1 $\Leftrightarrow$ add a particle (magnon)

$$S_j^z = -S + n_j$$

magnetization per site

$$m = \langle S_j^z \rangle = -S + \langle n_j \rangle = -S + \nu$$

zero magnetization (ground state of antiferromagnet)

$$m = 0 \qquad \nu = S$$

fractional filling if and only if $S$ is half-odd-int

# Why Haldane Gap?

Standard(?) view:

topological term of the O(3) non-linear sigma model present only for half-odd-integer spin *S*

Intuitive(?) view:

half-odd-integer spin *S*: fractional (1/2+integer) filling

integer spin *S*: integer filling → can be "trivial" insulator

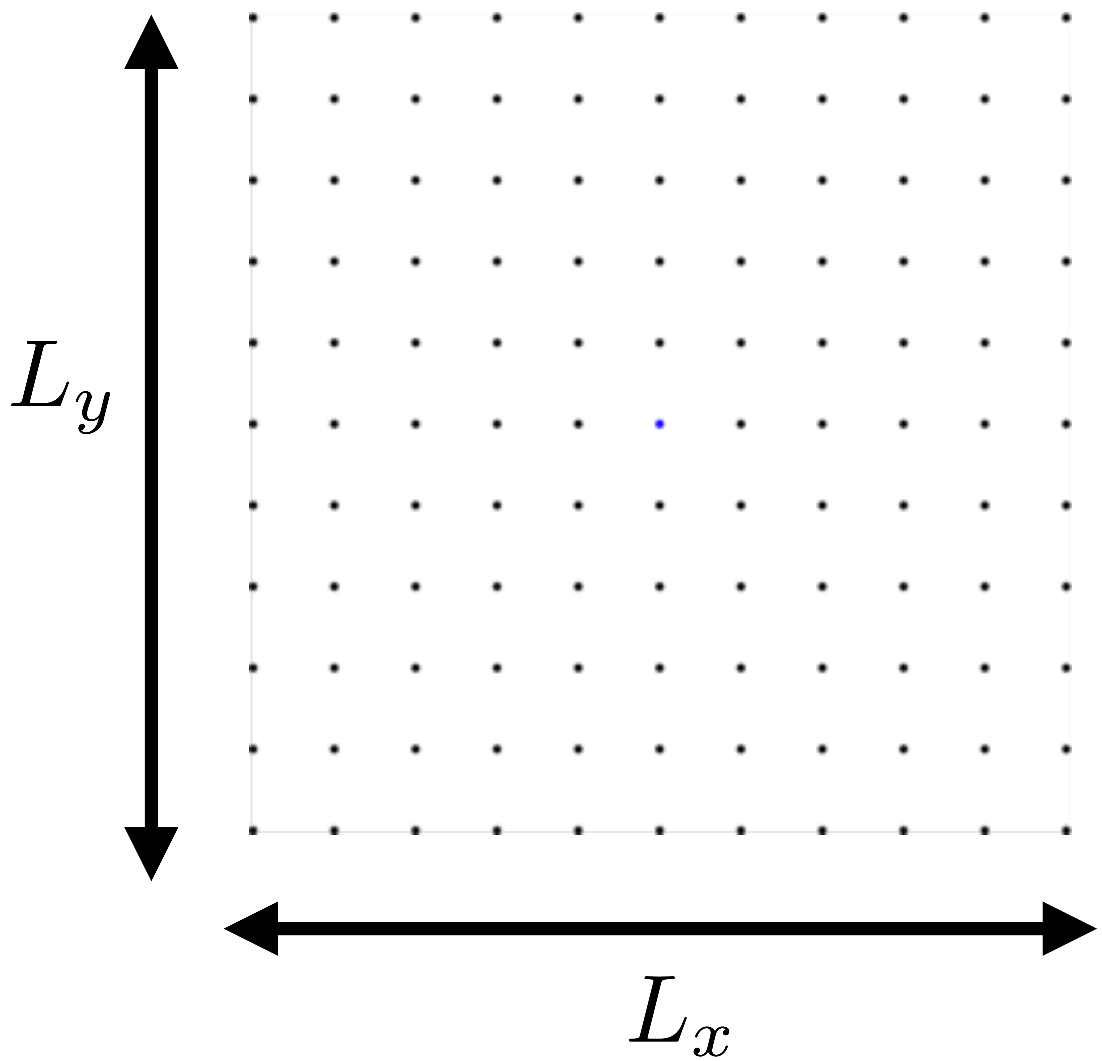
naturally obtained by generalizing the LSM theorem to many particle systems [Yamanaka-MO-Affleck 1997]

$$m = \langle S_j^z \rangle = -S + \langle n_j \rangle = -S + \nu$$

zero magnetization (ground state of antiferromagnet)

$$m = 0 \qquad \nu = S$$

# Higher Dimensions?



LSM twist in *x* direction

$$\mathcal{U}_x = \exp\left(\frac{2\pi i}{L_x}\sum_{\vec{r}} x n_{\vec{r}}\right)$$

$$\vec{r} = (x, y) \in \mathbb{Z}^2$$

Energy gain due to the twist $O(\frac{1}{{L_x}^2}) \times L_x L_y = O(\frac{L_y}{L_x})$

Not small….?!

# Anisotropic Limit

LSM variational argument works, if $L_y/L_x \to 0$

while $L_x, L_y \to \infty$, as already pointed out in LSM(1961)

In two dimensions we consider a square lattice of $N$ sites in the $x$-direction and of $M = O(N^\nu)$ sites in the $y$-direction, where $0 < \nu < 1$. The Hamiltonian is assumed cyclic in the sense that

$$\mathbf{S}_{n,\,M+1} = \mathbf{S}_{n,\,1} \tag{B-25a}$$

and

$$\mathbf{S}_{N+1,\,m} = \mathbf{S}_{1,\,m}\,, \tag{B-26}$$

i.e., the lattice is wrapped on a torus. We take for the operator $\mathcal{O}^k$,

$$\mathcal{O}^k = \exp\left(ik \sum_{n=1}^{N} \sum_{m=1}^{M} nS^z_{n,m}\right). \tag{B-27}$$

This operator twists the direction of all spins with the same $x$-coordinate by the same amount. $\Psi_k$ is constructed and its orthogonality to the ground state is proved precisely as in one dimension. Instead of (B-24), one now has

$$\langle \Psi_k \mid H \mid \Psi_k \rangle \leqq E_0 + (2\pi^2/N^{1-\nu}); \tag{B-28}$$

so again there is no energy gap. Because the excitation energy of exact low-lying states should not depend on the shape of the entire lattice, there should be no energy gap for a lattice of $N \times N$ sites either. The particular state $\Psi_k$ is unfortunately not sufficiently like an exact low-lying excited state to give this result.

A similar extension to three dimensions is obvious.

But is this really 2D limit?

Can we show LSM for isotropic 2D limit?

# Vector Potential: U(1) Gauge Field

Global U(1) symmetry in Quantum Mechanics
enhanced to U(1) gauge symmetry

$$\psi(\vec{r}) \to \psi(\vec{r})e^{i\theta} \quad \Longrightarrow \quad \psi(\vec{r}) \to \psi(\vec{r})e^{i\theta(\vec{r})}$$

Replace derivatives by "covariant derivative"

$$\vec{\nabla}\psi(\vec{r}) \quad \Longrightarrow \quad \left(\vec{\nabla} - i\vec{A}(\vec{r})\right)\psi(\vec{r})$$

$$\psi(\vec{r}) \to \psi(\vec{r})e^{i\theta(\vec{r})}$$

$$\vec{A}(\vec{r}) \to \vec{A}(\vec{r}) + \vec{\nabla}\theta(\vec{r})$$

covariant derivative

$$\left(\vec{\nabla} - i\vec{A}(\vec{r})\right)\psi(\vec{r})$$

is gauge invariant

# Meaning of Covariant Derivative

$$\partial_j \psi(\vec{r}) = \lim_{\delta \to 0} \frac{\psi(\vec{r} + \delta \vec{e}_j) - \psi(\vec{r})}{\delta}$$

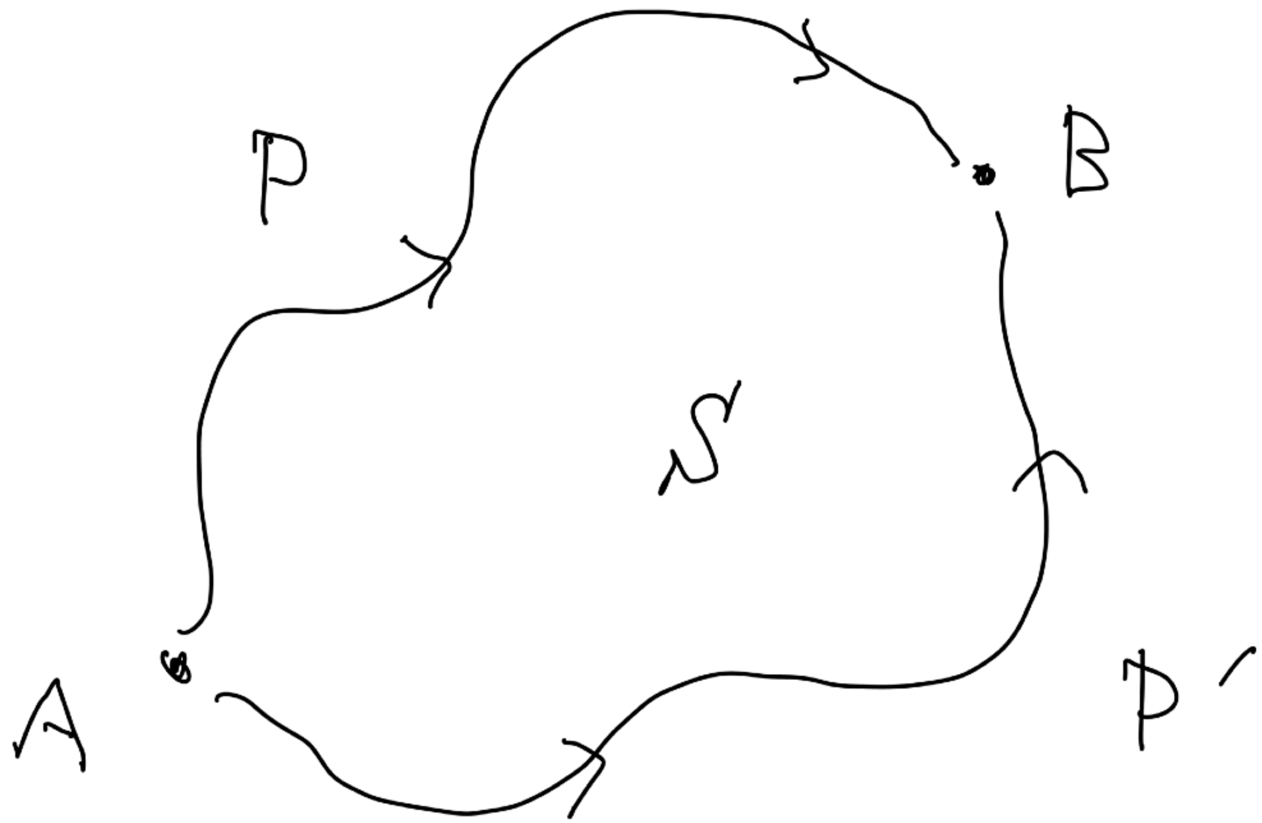
$$(\partial_j - iA_j)\,\psi(\vec{r}) = \lim_{\delta \to 0} \frac{\psi(\vec{r} + \delta \vec{e}_j) - \boxed{e^{i\vec{A}(\vec{r})\cdot\delta\vec{e}_j}\psi(\vec{r})}}{\delta}$$

"parallel transport"

Even when there were no vector potential initially, we can introduce a non-zero vector potential by a gauge transformation = local change of the phase

Before comparing wavefunctions at two points, we need the corresponding phase change ("parallel transport")

# Path Integral




extra phase

$$\exp\left(i\int_P \vec{A}(\vec{r})\cdot d\vec{r}\right)$$

due to the parallel transport along the path

$$\exp\left(i\int_P \vec{A}(\vec{r})\cdot d\vec{r} - \int_{P'} \vec{A}(\vec{r})\cdot d\vec{r}\right) = \exp\left(i\oint_{\partial S} \vec{A}(\vec{r})\cdot d\vec{r}\right)$$

$$\oint_{\partial S} \vec{A}(\vec{r})\cdot d\vec{r} = \int_S \mathrm{rot}\vec{A}\cdot d\vec{n}$$ Stokes' theorem

# Gauge Invariance

$\vec{B} = \mathrm{rot}\vec{A}$ ("curvature" = magnetic field) is gauge invariant

$$\mathrm{rot}\vec{A}' = \mathrm{rot}\left(\vec{A}' + \vec{\nabla}\theta\right) = \mathrm{rot}\vec{A}$$

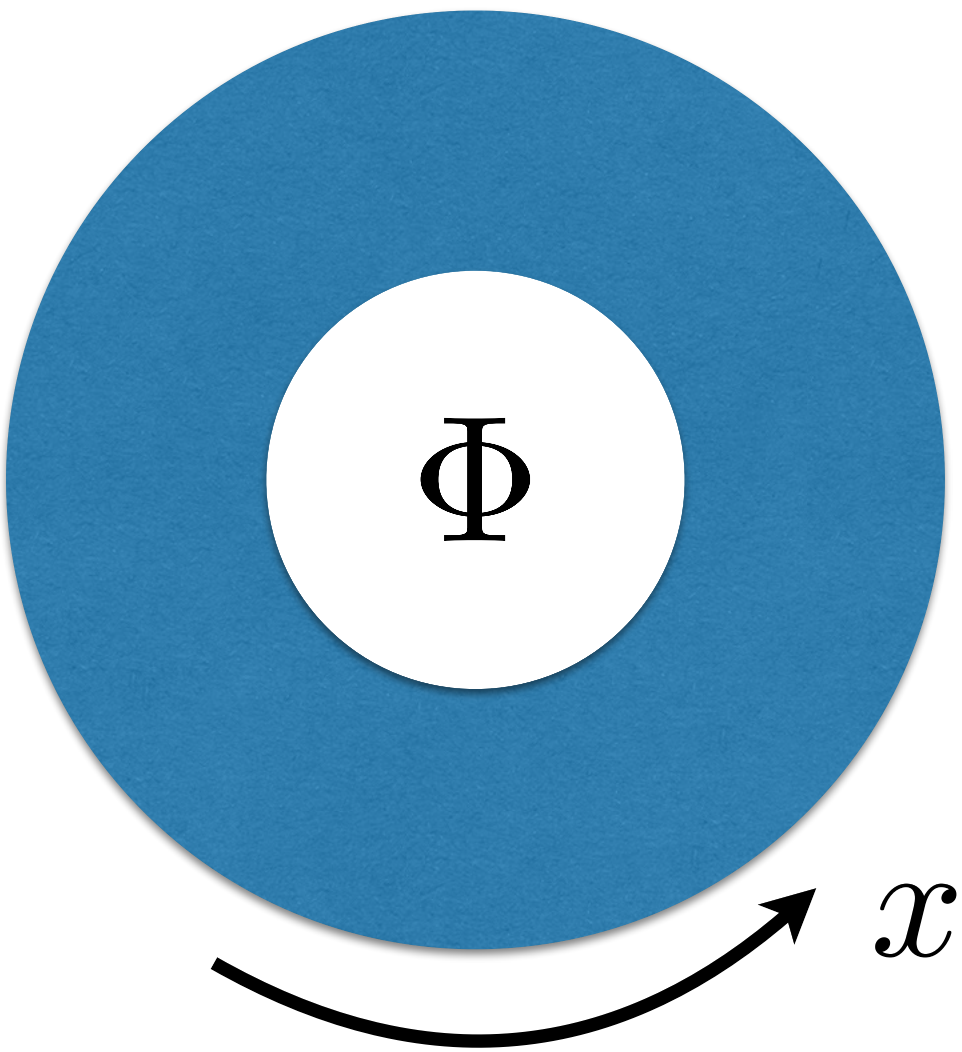
$$\oint_{\partial S} \vec{A}(\vec{r}) \cdot d\vec{r} = \int_S \mathrm{rot}\vec{A} \cdot d\vec{n} = \int_S \vec{B} \cdot d\vec{n} = \Phi(S)$$

phase difference = magnetic flux through the enclosed area

Only the gauge-invariant magnetic (and electric) field is physical

Vector potential has a gauge ambiguity and must be unphysical (just a mathematical trick) — right?

# Aharonov-Bohm Effect




particles do not touch
the magnetic field directly
$\Rightarrow$ no effect within classical mech

**But quantum interference is still affected $\Rightarrow$**

**Aharonov-Bohm effect**

Quantum system defined on the annulus does depend on the flux, except when the Aharonov-Bohm phase is

$$\Phi = 2\pi \times \text{integer}$$

# Unit Flux Quantum

I have implicitly chosen the units so that

$$\hbar = 1 \qquad\qquad e = 1$$

Covariant derivative ⇔ kinetic momentum

$$\left(-i\hbar\vec{\nabla} - e\vec{A}(\vec{r})\right)\psi(\vec{r})$$

$$\exp\left(i\frac{e}{\hbar}\oint_{\partial S}\vec{A}(\vec{r})\cdot d\vec{r}\right) = \exp\left[2\pi i\frac{\Phi(S)}{\Phi_0}\right]$$

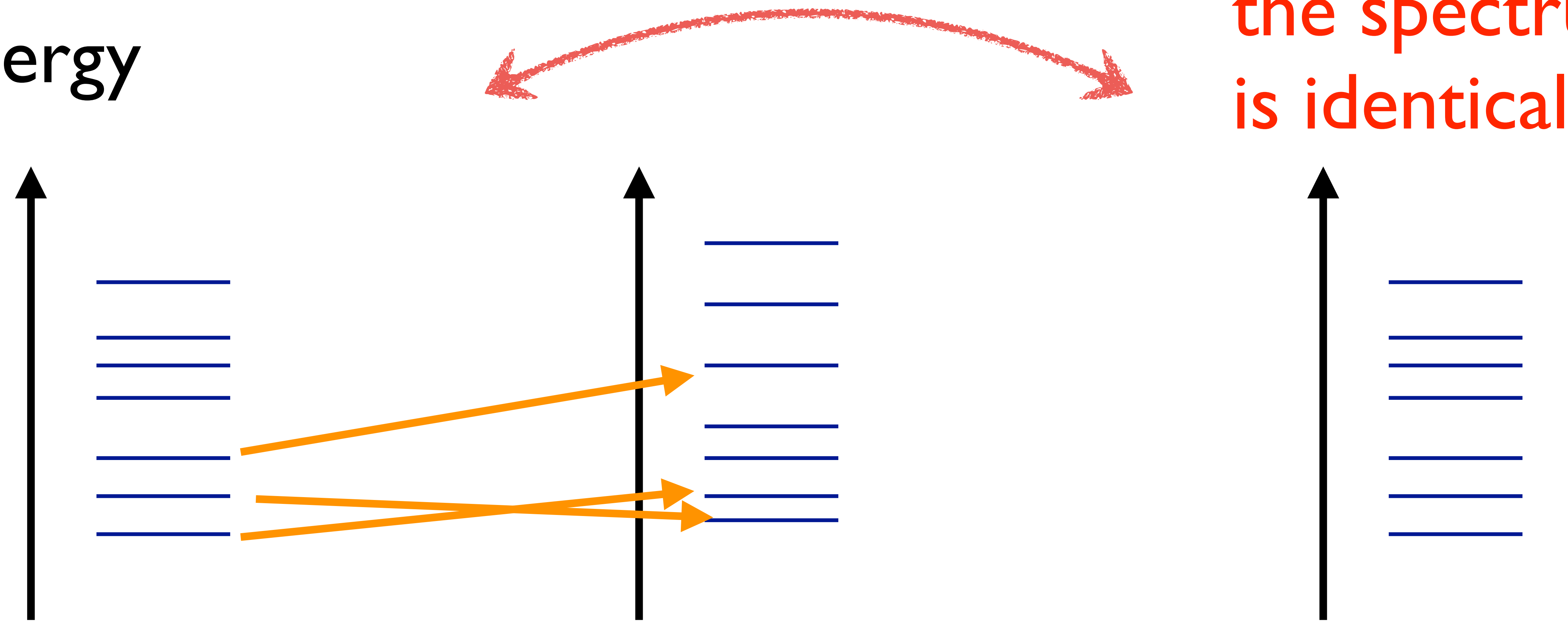
$$\Phi_0 = \frac{h}{e} = 4.136\times 10^{-15}\ \mathrm{Wb}$$

(twice the "unit flux quantum" commonly used in superconductivity literature)

# Spectrum of the Hamiltonian

Energy

the spectrum is identical!



$\Phi = 0$

generally depends on Φ (AB effect)

$\Phi = 2\pi (= \Phi_0)$

Nevertheless $\mathcal{H}(\Phi = 2\pi) \neq \mathcal{H}(\Phi = 0)$

# Large Gauge Transformation

If the Aharonov-Bohm flux is an integral multiple of the unit flux quantum it can be eliminated by a topologically nontrivial ("large") gauge transformation

$$\psi(\vec{r}) \to \psi(\vec{r})e^{i\theta(\vec{r})} \qquad \theta(\vec{r}) = 2\pi\frac{x}{L_x}$$

phase is multivalued but wavefunction is unique

For a many-body Hamiltonian on a lattice

$$\mathcal{H}(\Phi = 2\pi) = U_x{}^{-1}\mathcal{H}(\Phi = 0)U_x$$

$$U_x = \exp\left(\frac{2\pi i}{L_x}\sum_{\vec{r}} x n_{\vec{r}}\right)$$

identical to "LSM twist" operator!

# “Anomalous Symmetry”

Spinless Hubbard in *d* dimensions have

- U(1) symmetry (particle # conservation)
- Translation symmetry

as exact symmetries.

However, U(1) gauge transformation together with the translation produces an extra phase factor

$$U_x{}^{-1} T_x U_x = T_x \exp\left( \frac{2\pi i}{L_x} \sum_{\vec{r}} n_{\vec{r}} \right) = T_x \exp\left( 2\pi i L_y \nu \right)$$

corresponding to

“mixed ’t Hooft anomaly” in field theory

# LSM in arbitrary dimensions

LSM 1961, Affleck-Lieb 1985, M.O.-Yamanaka-Affleck 1997, M. O. 2000, Hastings 2004,…

Periodic (translation invariant) lattice ⇒ unit cell

**U(1) symmetry ⇒ conserved particle number**

$\nu$ : number of particle per unit cell (filling fraction)

$\nu = p/q \quad \Rightarrow$

- system is gapless

OR

- gapped with q-fold degenerate ground states

~~gapped with unique ground state~~

"ingappability"

must be in a nontrivial phase!

# Recent Developments

nature physics

ARTICLES

PUBLISHED ONLINE: 14 APRIL 2013 | DOI: 10.1038/NPHYS2600

**Topological order and absence of band insulators at integer filling in non-symmorphic crystals**

Siddharth A. Parameswaran[1], Ari M. Turner[2], Daniel P. Arovas[3] and Ashvin Vishwanath[1,4]*

Non-symmorphic lattice with "glide symmetry":
"effective unit cell" is half of the unit cell

$$\nu_{\text{eff}} = \frac{\nu}{2}$$

LSMOH-type restriction
even when $\nu \in \mathbb{Z}$

# Crystallographic Symmetries

## Filling constraints for spin-orbit coupled insulators in symmorphic and nonsymmorphic crystals

Haruki Watanabe[a], Hoi Chun Po[b], Ashvin Vishwanath[b,c], and Michael Zaletel[d,1]

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**Table 1.** Summary of $\nu_{\min}$ for elementary space groups

| | | Minimal filling | | | |
|---|---|---|---|---|---|
| ITC no. | Key elements | AI* | Ent† | Bbb‡ | Manifold name |
| 1 | (Translation) | 2 | 2 | 2 | Torus |
| 4 | $2_1$ | 4 | 4 | 4 | Dicosm |
| 144/145 | $3_1/3_2$ | 6 | 6 | 6 | Tricosm |
| 76/78 | $4_1/4_3$ | 8 | 8 | 8 | Tetracosm |
| 77 | $4_2$ | 4 | 4 | 4 | |
| 80 | $4_1$ | 4 | 4 | 4 | |
| 169/170 | $6_1/6_5$ | 12 | 12 | 12 | Hexacosm |
| 171/172 | $6_2/6_4$ | 6 | 6 | 6 | |
| 173 | $6_3$ | 4 | 4 | 4 | |
| 19 | $2_1$, $2_1$ | 8 | 4 | 8 | Didicosm |
| 24 | $2_1$, $2_1$ | 4 | 2 | 4 | |
| 7 | Glide | 4 | 4 | 4 | First amphicosm |
| 9 | Glide | 4 | 4 | 4 | Second amphicosm |
| 29 | Glide, $2_1$ | 8 | 4 | 8 | First amphidicosm |
| 33 | Glide, $2_1$ | 8 | 4 | 8 | Second amphidicosm |

*The minimal filling required to form a symmetric atomic insulator.
†$\nu_{\min}$ obtained in *Extension to 3D Symmorphic and Nonsymmorphic Crystals*. Bounds are not tight for nos. 19, 24, 29, and 33.
‡$\nu_{\min}$ obtained in *Alternative Method: Putting Sym-SRE Insulators on Bieberbach Manifolds*. All bounds are tight.

# LSM for Discrete Symmetry

Neither the LSM "slow twist" or U(1) flux insertion works for discrete symmetries, but the generalization of the LSM holds!

[1D]

MPS-based "proof" Chen-Gu-Wen 2011

Field-theory argument Fuji 2014

Mathematical proof Ogata-Tachikawa-Tasaki 2020

[2D and higher]

Po-Watanabe-Jian-Zalatel 2017, Else-Thorngren 2020

Watanabe-Po-Vishwanath-Zalatel 2015

Yao-M.O. 2021

# Example: XYZ model

"XYZ" spin model on the square lattice of size $L_1 \times L_2$

$$\mathcal{H} = \sum_{\langle \vec{r}, \vec{r}' \rangle} \left( J_X S^x_{\vec{r}} S^x_{\vec{r}'} + J_Y S^y_{\vec{r}} S^y_{\vec{r}'} + J_Z S^z_{\vec{r}} S^z_{\vec{r}'} \right)$$

On-site discrete symmetry of $Z_2 \times Z_2$ (dihedral sym.)
(π-rotation of spins about *x*, *y*, and *z* axes)
Lattice translation symmetry $T_1$, $T_2$

odd number of "spin 1/2" per unit cell
→ ground-state degeneracy or gapless spectrum

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**Anomalous domain wall condensation in a modified Ising chain**

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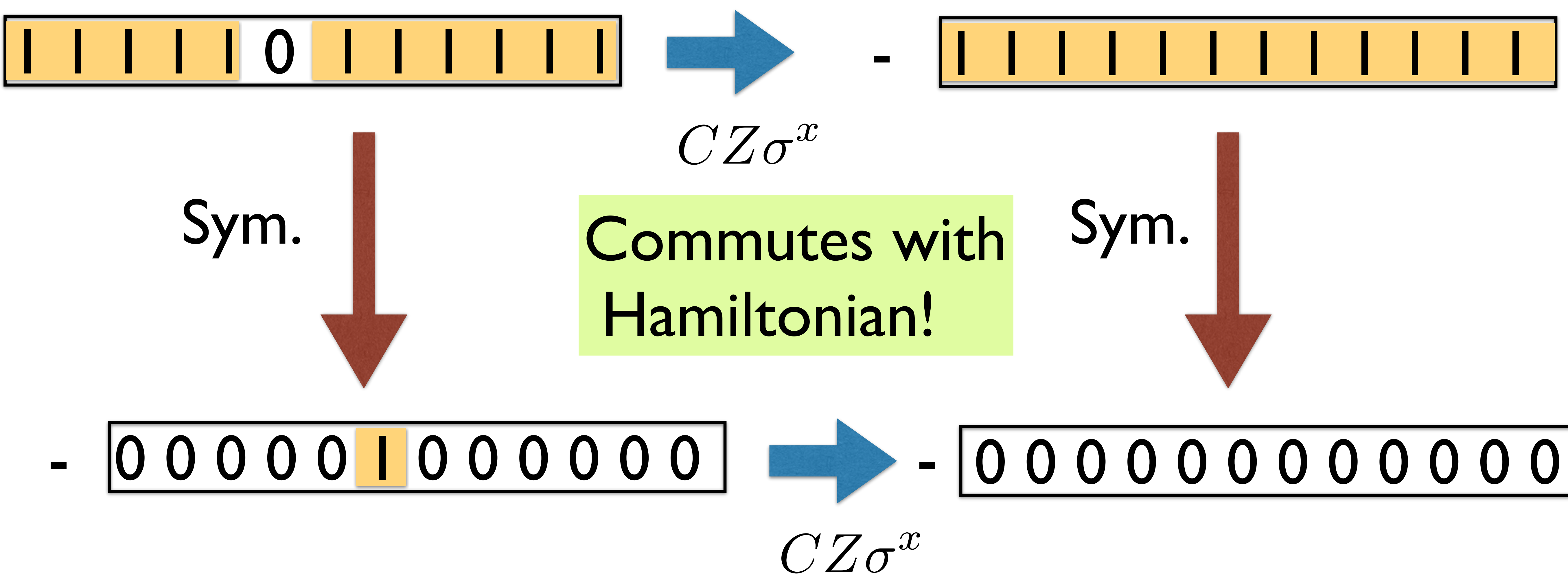
$$H = \sum_i CZ_{i-1,i+1}\sigma_i^x - \mu\sigma_i^z\sigma_{i+1}^z ,$$

$$CZ|\uparrow\uparrow\rangle = |\uparrow\uparrow\rangle$$
$$CZ|\uparrow\downarrow\rangle = |\uparrow\downarrow\rangle$$
$$CZ|\downarrow\uparrow\rangle = |\downarrow\uparrow\rangle$$
$$CZ|\downarrow\downarrow\rangle = -|\downarrow\downarrow\rangle$$

does not have the standard Z2 symmetry (spin flip)

# Anomalous Z2 Symmetry

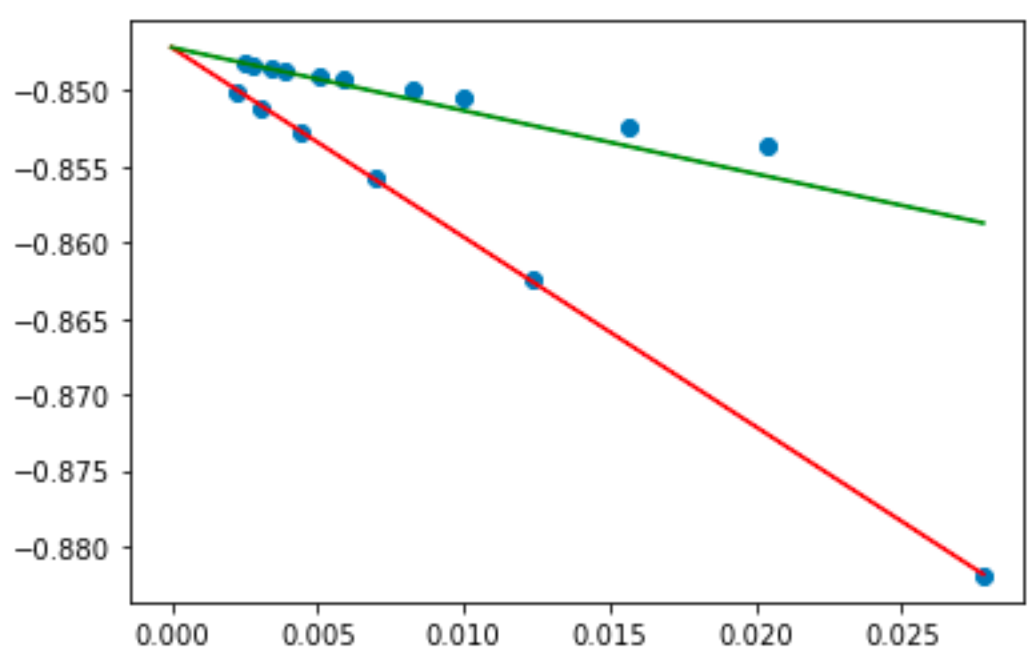
$$(-1)^{\#\mathrm{strings}} \bigotimes_i \sigma_i^x$$



# Consequences

The system is "ingappable" (gapless or g.s. degeneracy)


G.S. energy per site




$1/L^2$

Exact Diagonalization
with Python
up to $L$=21
consistent with
Conformal Field Theory
(quantum critical)

# Summary

- Metals are gapless and conducting, but this is rather nontrivial from statistical mechanics point of view
- Gaplessness (≒ quantum criticality, conductivity) is achieved either by fine-tuning or some "protection"
- Lieb-Schultz-Mattis (LSM)-type theorems provide powerful and general constraints, which often protect quantum criticality in translation invariant & charge conserving systems
- Active topic of current research with numerous generalizations