Resolving Berezinskii-Kosterlitz-Thouless Transition in 2D XY model with TNR+CFT

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[Submitted on 24 May 2021]

Resolving the Berezinskii-Kosterlitz-Thouless transition in the 2D XY model with tensor-network based level spectroscopy

Atsushi Ueda, Masaki Oshikawa

Berezinskii-Kosterlitz-Thouless transition of the classical XY model is re-investigated, combining the Tensor Network Renormalization (TNR) and the Level Spectroscopy method based on the finite-size scaling of the Conformal Field Theory. By systematically analyzing the spectrum of the transfer matrix of the systems of various moderate sizes which can be accurately handled with a finite bond dimension, we determine the critical point removing the logarithmic corrections. This improves the accuracy by an order of magnitude over previous studies including those utilizing TNR. Our analysis also gives a visualization of the celebrated Kosterlitz Renormalization Group flow based on the numerical data.

Subjects: **Statistical Mechanics (cond-mat.stat-mech)**
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just appeared in arXiv

Atsushi Ueda

Scientific Background on the Nobel Prize in Physics 2016

TOPOLOGICAL PHASE TRANSITIONS AND
TOPOLOGICAL PHASES OF MATTER

compiled by the Class for Physics of the Royal Swedish Academy of Sciences

Prototype: Berezinskii-Kosterlitz-Thouless Transition

Canonical model: 2D classical XY model

$$H_{XY} = -J \sum_{\langle ij \rangle} \cos(\theta_i - \theta_j)$$

"easy" to simulate by Monte Carlo?

Journal of the Physical Society of Japan **81** (2012) 113001 LETTERS
http://dx.doi.org/10.1143/JPSJ.81.113001

Large-Scale Monte Carlo Simulation of Two-Dimensional Classical XY Model Using Multiple GPUs

Yukihiro KOMURA* and Yutaka OKABE†

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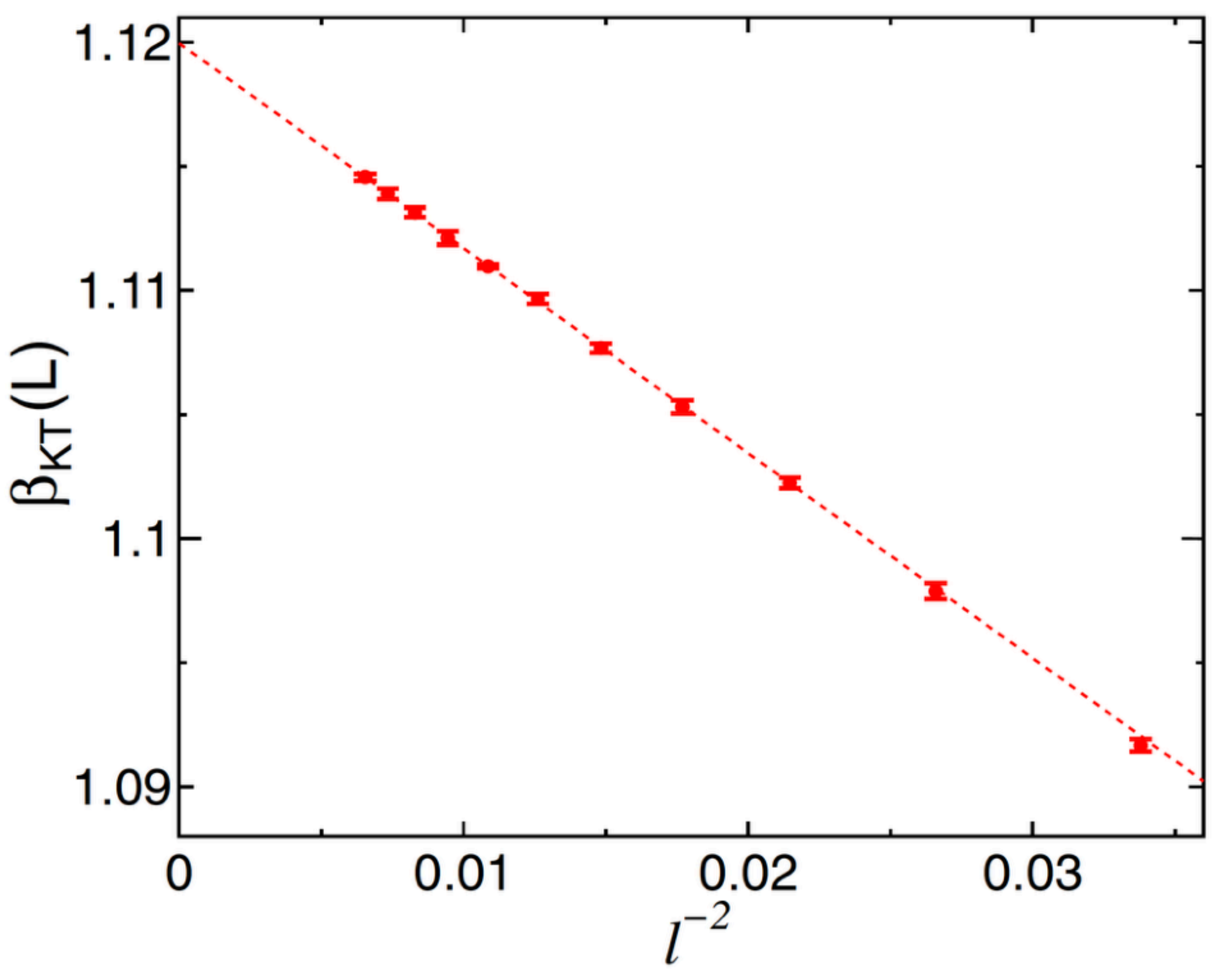
(Received August 27, 2012; accepted September 24, 2012; published online October 12, 2012)



finite-size scaling of T_c

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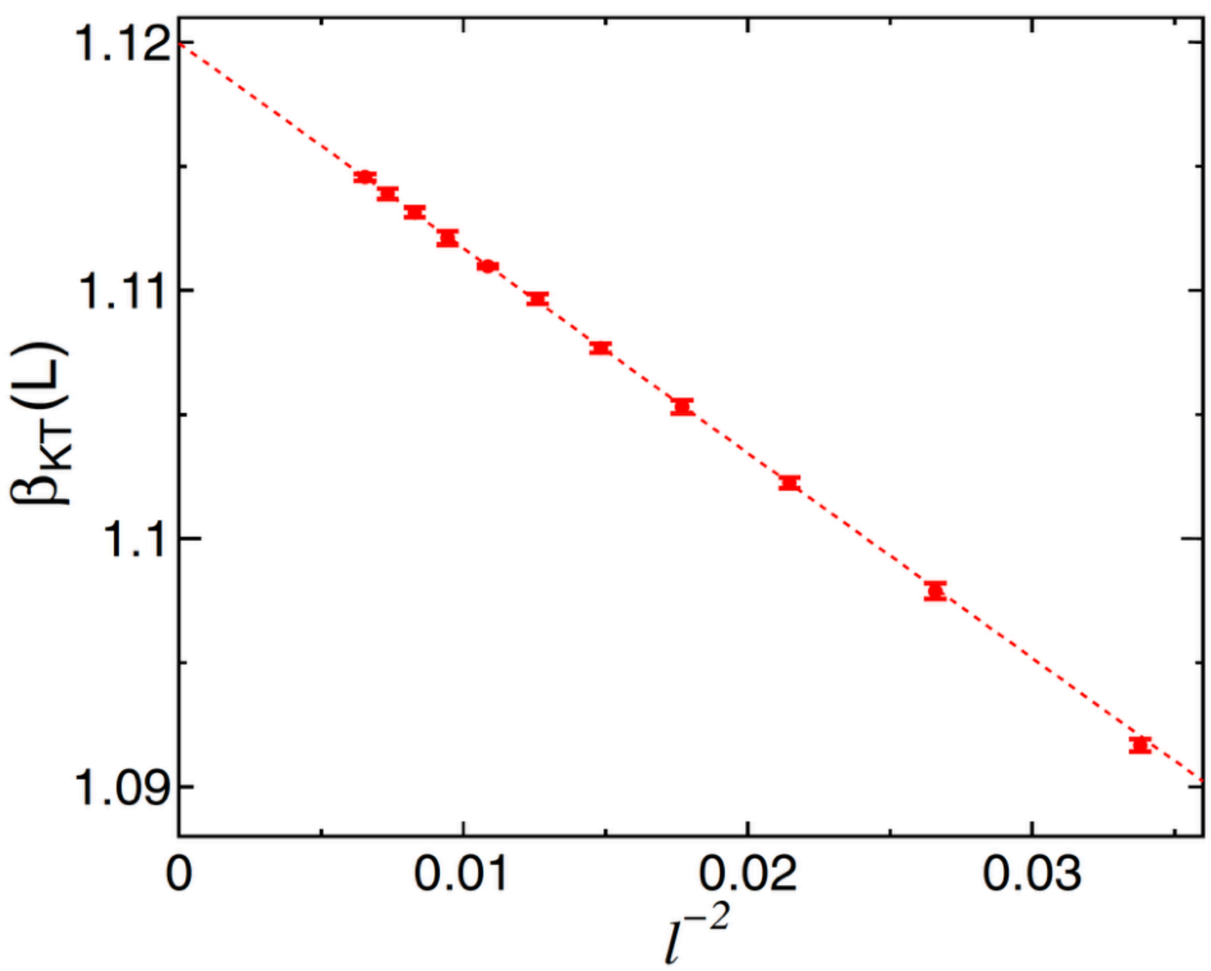

finite-size scaling of T_c

$$l = \ln bL$$

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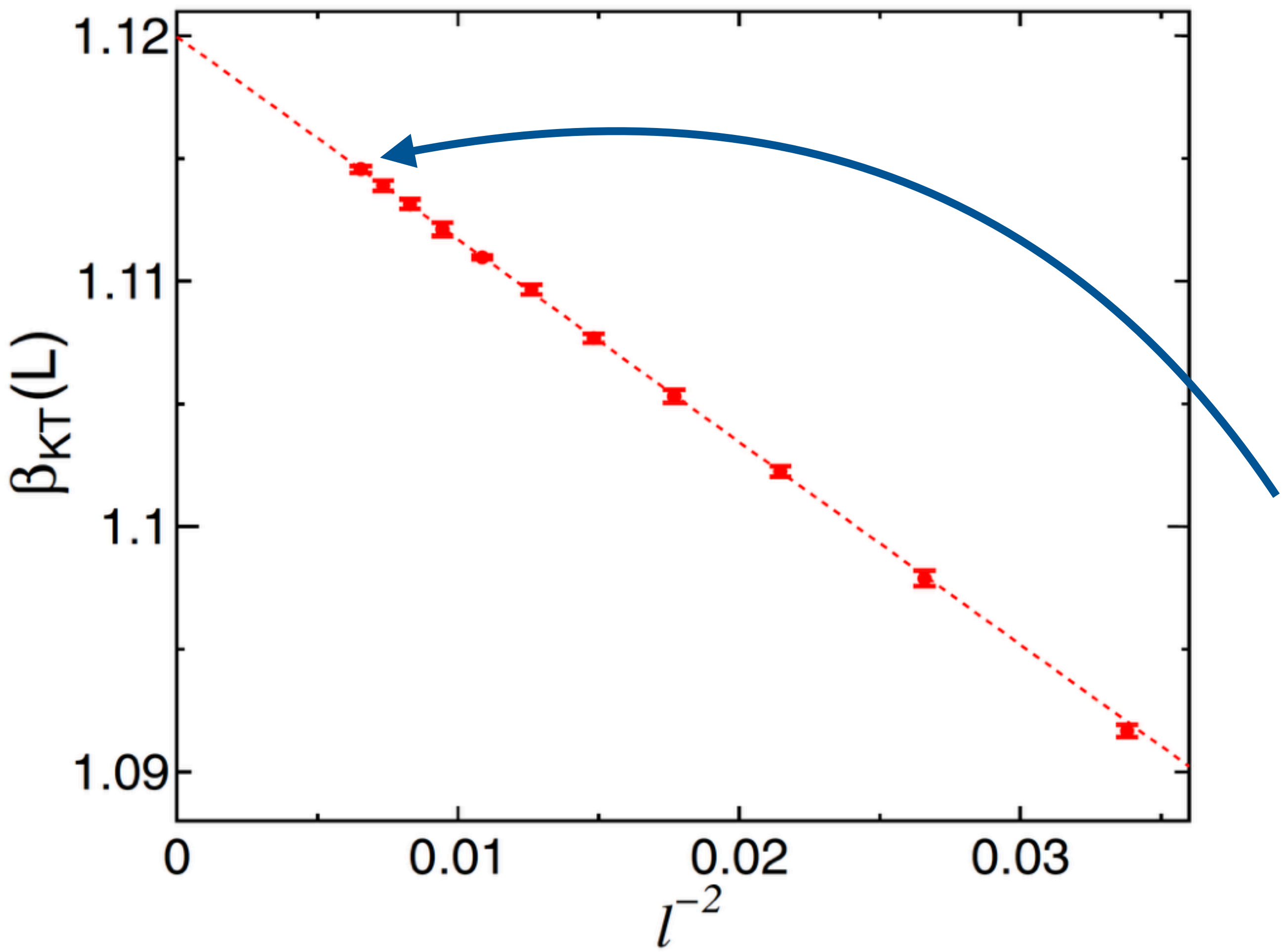
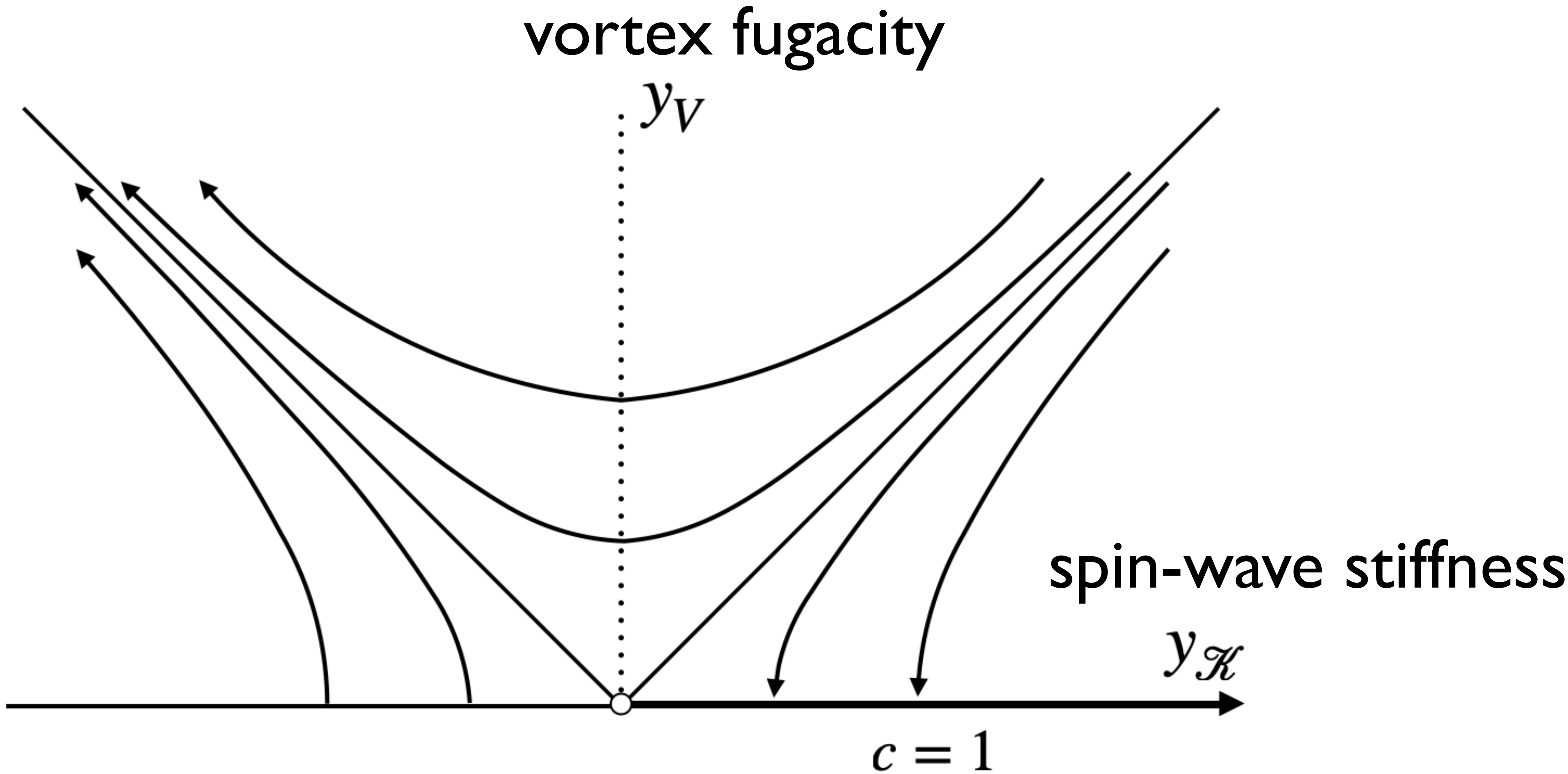


finite-size scaling of T_c

$$l = \ln bL$$

largest system:
L=65536
calculation using 256GPUs

Kosterlitz RG Flow



BKT transition: $y_V = y_{\mathcal{K}} = g$

slow decay

↓

log-corrections

$$\frac{dg}{dl} = -g^2$$

$$g \sim \frac{1}{l} \sim \frac{1}{\ln L}$$

BKT Transition in S=1/2 XXZ Chain

$$\mathcal{H} = \sum_j \left(S_j^x S_{j+1}^x + S_j^y S_{j+1}^y + \Delta S_j^z S_{j+1}^z \right)$$

BKT transition at Δ=1 (SU(2) symmetric point)

Effective theory in the vicinity of the BKT transition

$$\mathcal{L} = \mathcal{L}_{k=1}^{\mathrm{WZW}} + g \mathbb{J}^L \cdot \mathbb{J}^R + t \left(-\frac{1}{2} J_+^L J_+^R - \frac{1}{2} J_-^L J_+^R + J_z^L J_z^R \right)$$

$$y_V = g + t, y_{\mathcal{K}} = g - t$$

BKT transition ⇔ t=0 ⇔ SU(2) symmetry

Level Spectroscopy

Determination of the critical point from the finite-size spectrum [Okamoto-Nomura 1994]

BKT transition can be identified by SU(2) symmetry of the finite-size spectrum!!

State-operator correspondence in CFT

$$E_n - E_0 = \frac{2\pi}{L} \left(x_n + \sum_m c_{nnm} y_m L^{2-x_m} + \dots \right)$$

BKT transition ⇔

Energy levels form SU(2) singlet, triplet, …

1D S=1/2 XXZ vs 2D Classical XY

$$\mathcal{L} = \frac{1}{2\pi K}(\partial_\mu \phi)^2 - y_K (\partial_\mu \phi)^2 + y_V V$$

S=1/2 XXZ

K=1/2 (SU(2)$_1$ WZW)

$V \sim \cos 4\phi$

double vortex op.

SU(2) triplet (degenerate at BKT)

$n^x \sim \cos\theta, n^y \sim \sin\theta, n^z \sim \sin 4\phi$

$2\phi \to \phi$

Classical XY

K=2

$V \sim \cos 2\phi$

single vortex op.

$\cos 2\theta, \sin 2\theta, \sin\phi$

half-vortex op. (eigenstate under antiperiodic b.c.)

Kitazawa-Nomura 1998

Level Spectroscopy for 2D Stat Mech

Level spectroscopy has been developed for quantum 1D

1D quantum Hamiltonian ⇔

Transfer matrix for 2D stat mech

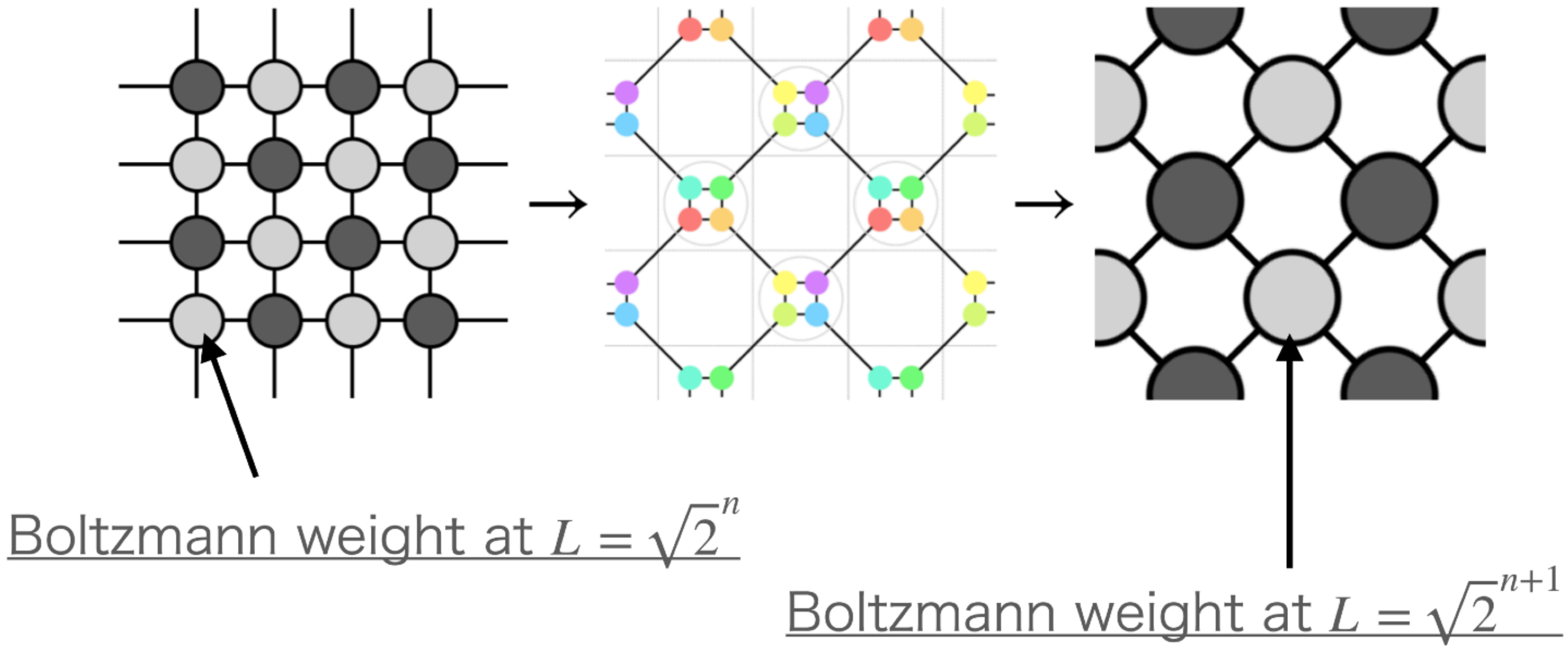
Continuous spin: series expansion of Boltzmann weight

$$e^{\beta \cos(\theta_i - \theta_j)} = \sum_{n=-\infty}^{\infty} e^{in(\theta_i - \theta_j)} I_n(\beta),$$

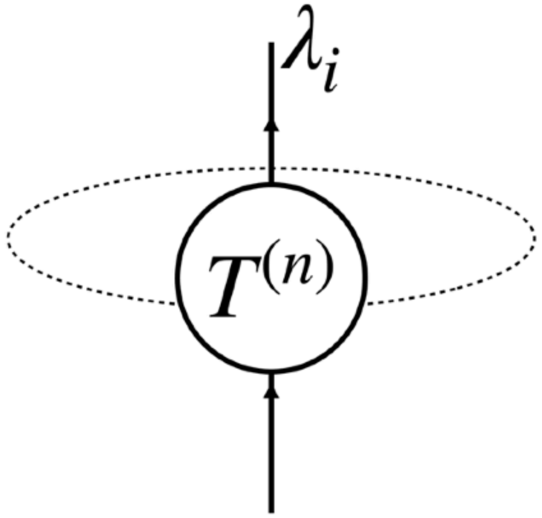
Transfer matrix still "too large" to be diagonalized

⇒ we utilize Tensor Network Renormalization

TNR Construction of Transfer Matrix



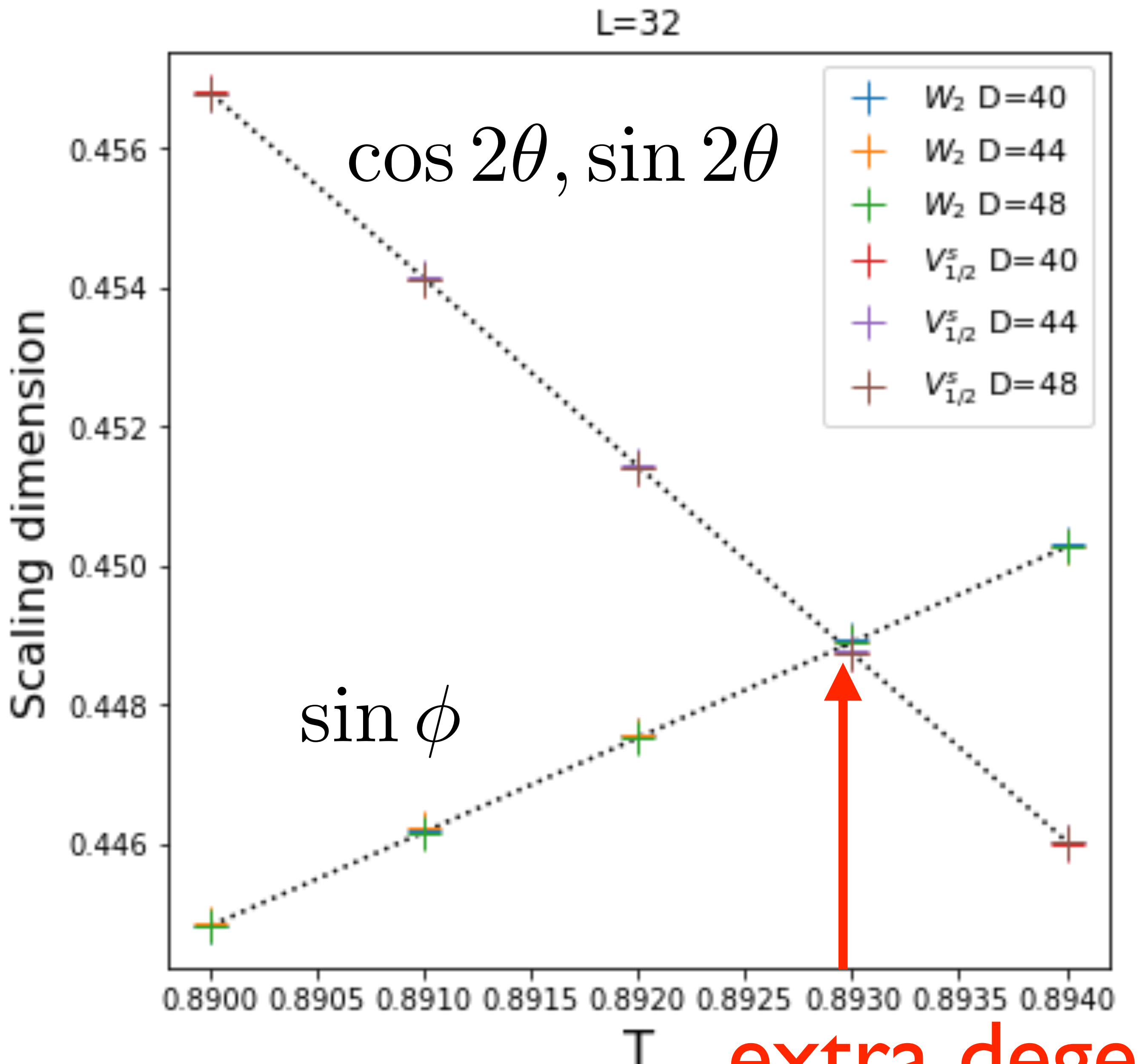
after n steps, a single tensor represents
a square block of linear size $L = \sqrt{2}^n$



contract horizontal indices
⇒ transfer matrix in vertical direction

$$\lambda_i = e^{-LE_i(L)}$$

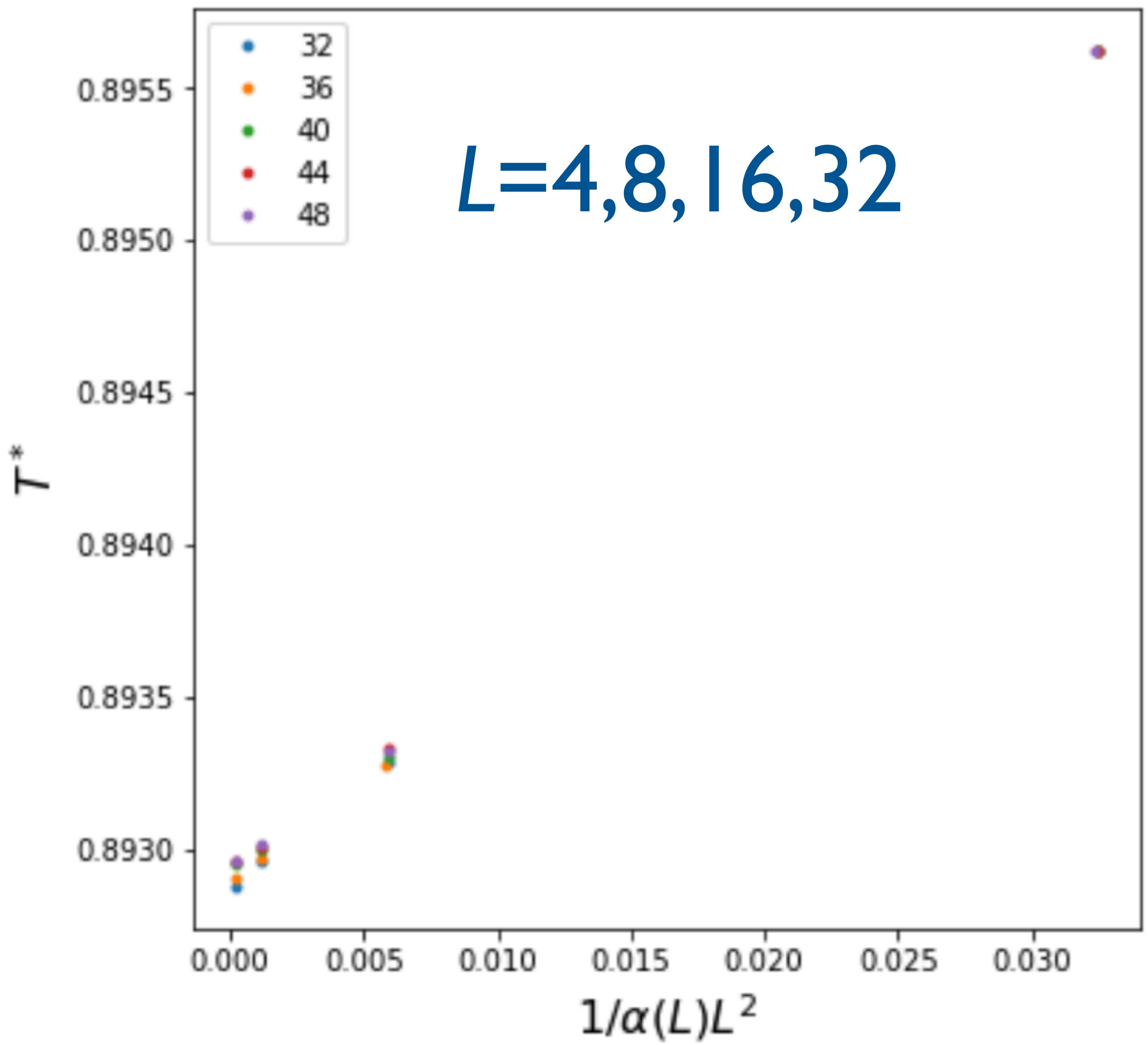
Level Crossing



This procedure eliminates logarithmic corrections to all orders in g

extra degeneracy
forming SU(2) triplet
$\sim$ BKT transition

Remaining Finite-Size Effect



Extrapolate to $L=\infty$

Level crossing point weakly depends on the system size *L*

Effect of irrelevant perturbations

$$T^2, \bar{T}^2, T\bar{T}, \ldots$$

T: holomorphic part of the energy-momentum tensor

$$T^* \sim T_c + \text{const.} \frac{1}{L^2}$$

Effect of Finite Bond-Dimension

Finite bond dimension *D* ⇔ finite "correlation length"

$$\xi_D \sim 0.3 D^{\kappa}$$

$$\kappa = \frac{6}{c\left(\sqrt{\frac{12}{c}}+1\right)}$$

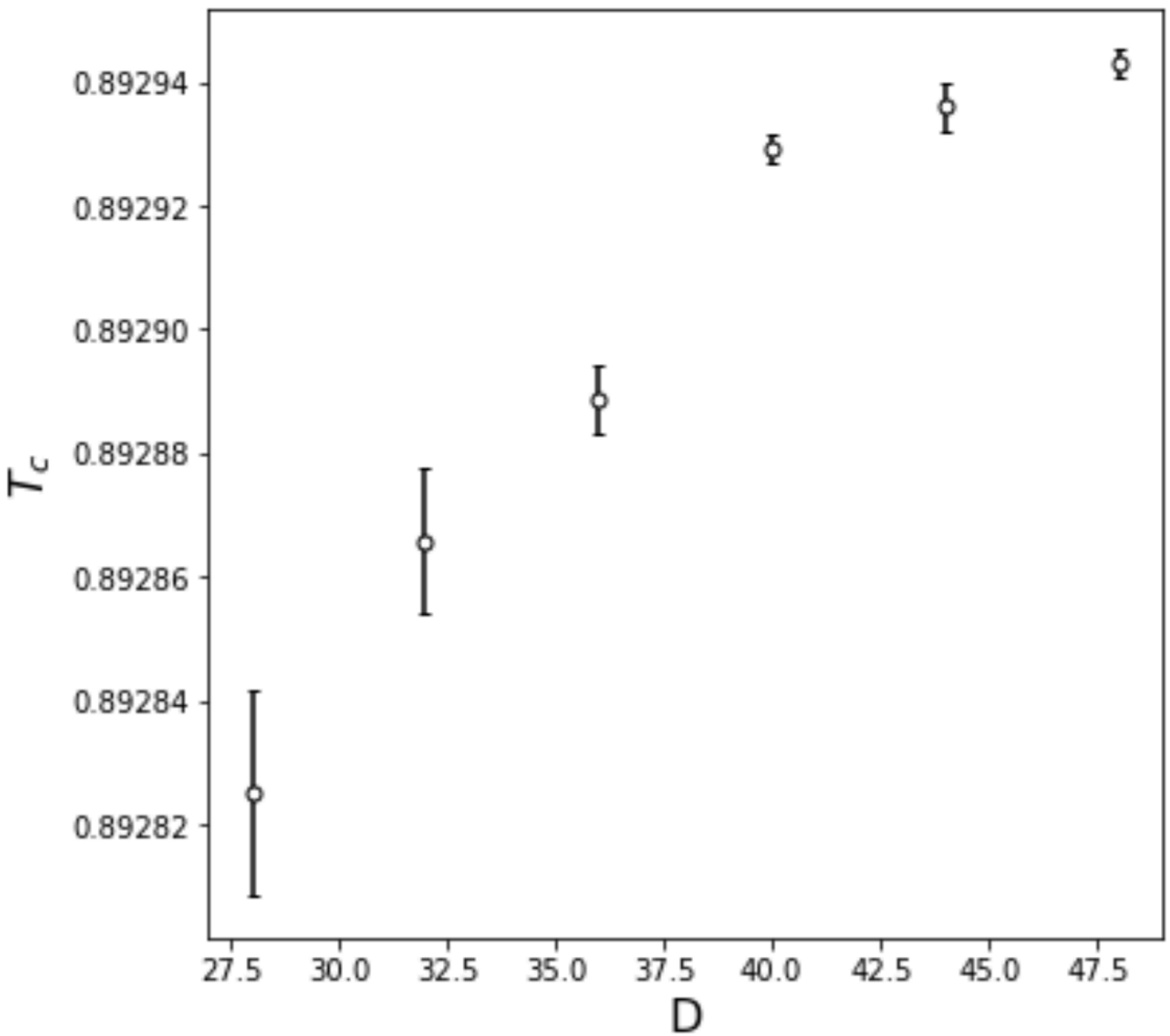
[Pollmann et al. 2008]

$\xi_D > L$ low-energy spectrum almost exact!

$\xi_D < L$ low-energy spectrum still reasonably accurate, but some error due to the finite *D*

cf.) "fixed point tensor" from TNR

Tc dependence on *D*



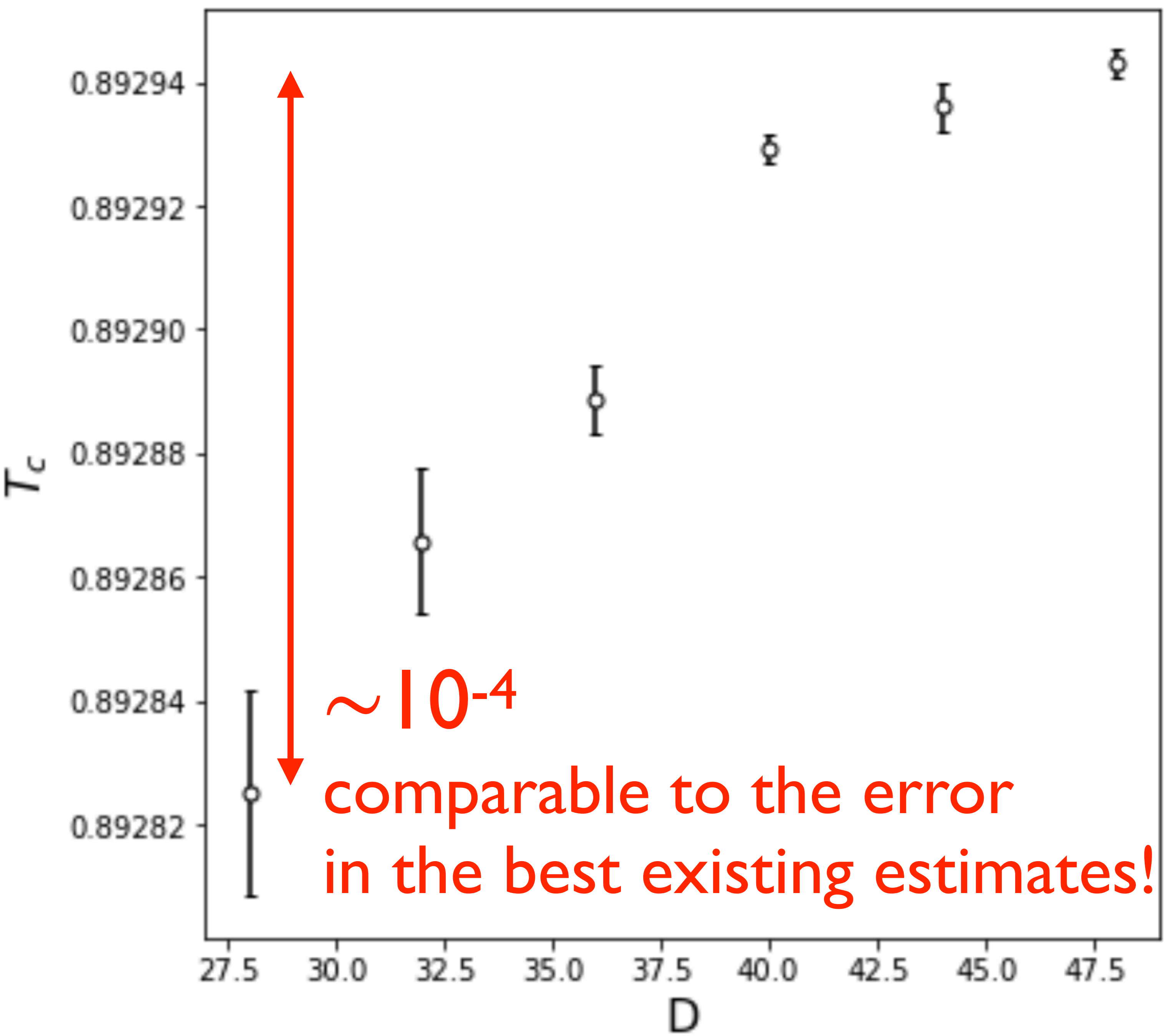
D=48 gives ξ～54

enough for up to *L*=32

D=28 gives ξ～26

too small for *L*=32
BUT….

Tc dependence on *D*



D=48 gives ξ～54

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too small for *L*=32
BUT….

Estimates of T_c

Monte Carlo(1979)[35]	0.89
Monte Carlo(2005)[36]	0.8929
Monte Carlo(2012)[37]	0.89289
Monte Carlo(2013)[38]	0.8935
Series expansion(2009)[39]	0.89286
HOTRG(2014)[40]	0.8921
VUMPS(2019)[41]	0.8930
HOTRG(2020)[42]	0.89290(5)
present work	0.892943(2)

TABLE I. Comparison of the estimated critical temperature of the 2D classical XY model.

Extraction of Couplings

Energy levels vs marginal perturbations $E_n - E_0 = \frac{2\pi}{L} x_n$

$$x_{W_{\pm 2}} = \frac{1}{2} - \frac{y_\mathcal{K}}{4} + \frac{1}{4} y_V^2,$$

[Lukyanov 1998]

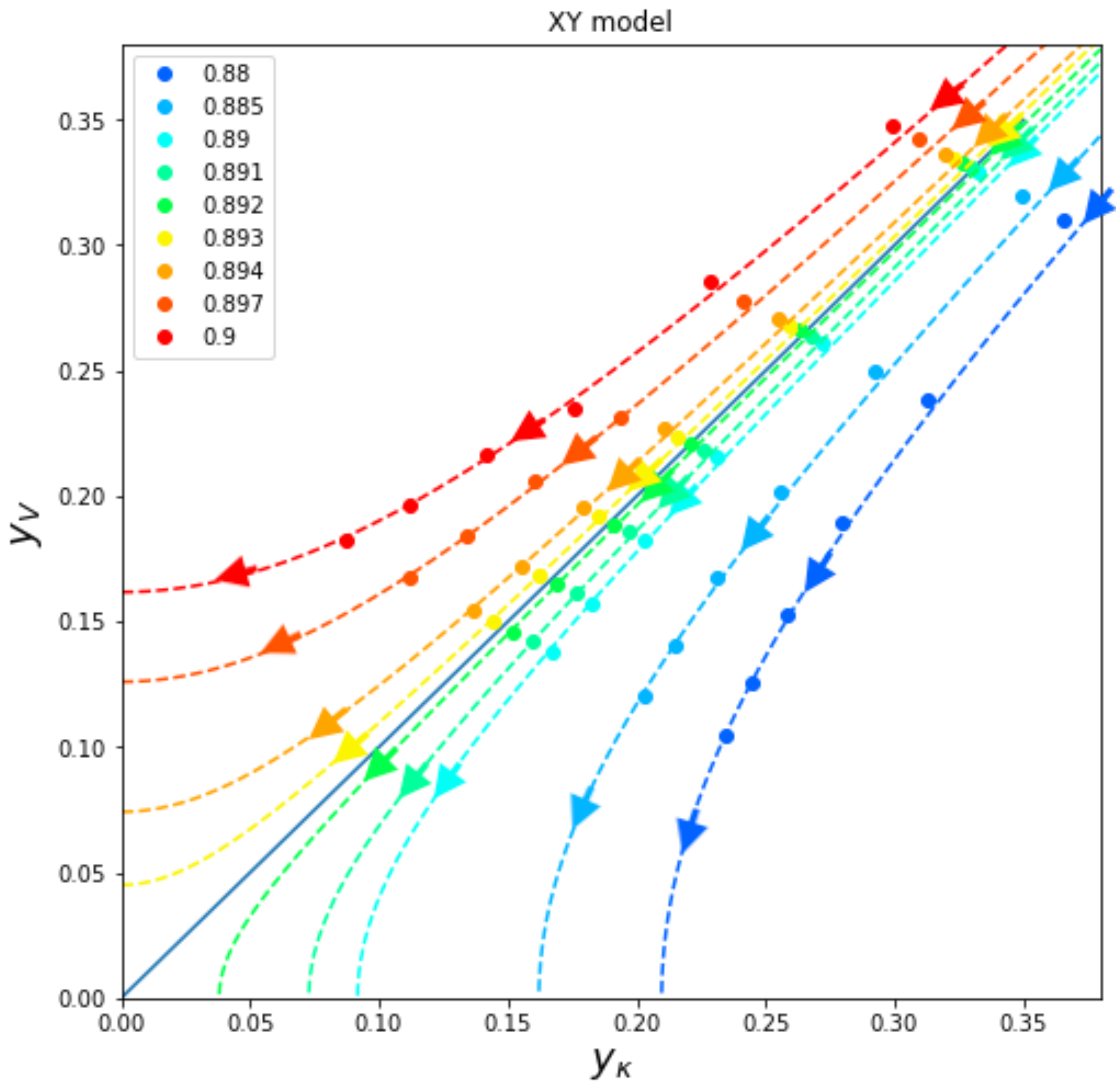
$$x_{V^s_{1/2}} = \frac{1}{2} + \frac{y_\mathcal{K}}{4} - \frac{y_V}{2} + \frac{1}{8}(y_\mathcal{K}^2 + 2y_\mathcal{K}y_V - y_V^2),$$

$$x_{V^c_{1/2}} = \frac{1}{2} + \frac{y_\mathcal{K}}{4} + \frac{y_V}{2} + \frac{1}{8}(y_\mathcal{K}^2 - 2y_\mathcal{K}y_V - y_V^2),$$

⇒ estimate y_K & y_V from the finite-size energy levels

Less accuracy than T_c, but then can apply to larger systems (up to L=512)

Visualization of Kosterlitz RG Flow!



Conclusions

TNR + Level Spectroscopy (finite size scaling of CFT) allows

- super accurate determination of BKT critical point
- visualization of Kosterlitz RG flow by extraction of running coupling constants from the spectrum

for continuous valued 2D classical spin system such as XY model

Future: extension/application to more nontrivial systems & unknown physics

(stay tuned!)